Krzysztof Targiel
Department of Operations Research
University of Economics in Katowice

# SCHEDULING RESOURCE CONSTRAINED PROJECT PORTFOLIOS WITH THE PRINCIPLES OF THE THEORY OF CONSTRAINTS[1]

## Introduction

Contemporary practice of project management is evolving towards accumulation of all projects in one organizational unit. This tendency manifests itself in creating a project management offices (PMO - Project Management Office). Their tasks include planning implementation and control of all projects within the organization.

As stressed in the PMBoK, one of the key properties of the PMO is to share and coordinate resources across all projects administered by the office (PMBOK 2004, p. 18). Updating the portfolio by adding new projects, they must make schedules, that all resources are best used. On the other hand, each project must have access to needed resources. This task can be reduced to the problem of scheduling with limited resources.

The problem of scheduling the project portfolio can be seen as a single project scheduling, in which there are many activities related to inconsistent graphs activities. However, they are linked to the need to use common limited resources. In this light, the problem becomes the selection of new projects in the portfolio, and thus the modification of existing schedules. The existing schedules should be forced into a new job using the available free resources.

Work attempts to formalize the principles of the Theory of Constraints for use in the project portfolio scheduling with limited resources. In particular, it will propose a identification of strategic resource, and then simplified process of scheduling by proposed heuristic.

[1] The research is supported by NCN grant no. NN 111 267138

## 1. Resource Constrained Project Scheduling Problem

Resource Constrained Project Scheduling Problem (RCPSP) is widely considered in the literature. The problem is solved using metaheuristics, such as ant systems (Merkle, Middenforf, Schmeck 2002), swarms of particles (Jarboui, Damak, Siarry, Rebai 2008). They are used with the taboo search and simulated annealing (Mika, Waligora, Węglarz 2005), local search and genetic and evolutionary algorithms. It also proposes specific heuristics. Examples are the work of Neumann and Zimmermann (Neumann, Zimmermann 2000) or Tormosa and Lova (2003). They are used as optimization method. The work Ayala and Artigues (2010) used the method of linear programming. These articles are examples of recent work presenting ways to solve RCPSP. Query executed on the end of 2010 showed 458 entries for the problem.

## 2. Theory of Constraints

Theory of Constraints (TOC) was proposed by Eliyahu Goldratt (1997). The concept of the theory is based on two fundamental postulates:

- Each system must have at least one constraint
- The existence of constraints makes it possible to develop

Author proposed the method of applying the Theory of Constraints which consists of five iterative steps (Goldratt 1997):

1) identify constraints of the system
2) decide how to exploit the constraints
3) subordinate all others to the decisions on exploiting the constraints
4) elevate the constraints
5) by returning to Step 1, determine a new constraint

The basic element is the identification of constraints. If it is determined you can develop a method of disposal. This task should be given to all the action. Removal of this restriction allows the achievement of the organization (system) more efficient, which means that the system is based on another limitation. So the whole procedure should be repeated.

The use of the Theory of Constraints in Project Management was proposed by Goldratt (2007). The work is concerned the presentation of the Critical Chain method, streamlines the implementation of individual projects. Author also draws attention to the specificity of portfolio management, but here is restricted to introducing the concept of a strategic resource, which is a constraint in the company being considered system.

Strategic resource is a 'bottleneck' - which determines the possibility of implementation that has the company. It is constraint that dictates the rhythm of the work of other resources. Hence, the bandwidth determines the organization. For an organization to be effective resource that must be used at 100%.

To be able to use the TOC, you will need is the ability to identify strategic resource. To solve this problem was proposed as described in the next chapter heuristics.

## 3. Proposed method

Application of TOC philosophy in scheduling, significantly reduces the space of possible exploration. We focus only on optimal, in the sense of maximum utilization, scheduling the strategic resource. There is only a problem of defining what is a strategic resource. This paper proposes a heuristic algorithm, which allows you to define a strategic resource and then find schedule for new project.

We adapt Artigues' model (Artigues, Michelon, Reusser 2003) to the multi-project environment. We use standard notation for scheduling RCPSP problems, proposed by Brucker (Brucker et al. 1999), where:

– $Q$ – set of projects in portfolio
– $V^q$ – is set of activities of $q$-th project $q \in Q$
– $q$ – is project index
– $q + 1$ – is new project
– $\Re^\rho$ – set of renewable resources
– $R_k^\rho$ – availability renewable $k$-th resource – $k \in \Re^\rho$
– $p_j^q$ – processing time of activity $j$ in $q$-th project

RCPSP problem is to find schedule for each $q$-th project as vector:

$$S^q = \left(S_0^q, \ldots, S_j^q, \ldots, S_{n+1}^q\right)$$

where:

$S_j^q$ $j = 0, 1, \ldots, n, n+1$ – is the starting point of activity $j$ in $q$-th project.

We add two points in time for each $q$-th project:
$S_0^q$ – is moment when $q$-th project starts
$S_{n+1}^q$ – is moment when $q$-th project is finished

To simplify the notation we introduce also the end times for each $j$-th activity and each project $q$:

$$C^q = \left(C_0^q, \ldots, C_j^q, \ldots, C_{n+1}^q\right)$$

where:

$C_j^q$ $j = 0,1, \ldots, n, n+1$ – is the time of completion of activity $j$

We will define set of activities ongoing at time $t$ for each project $q$:

$$A(S^q,t) = \left\{ j \in V^q \mid S_j^q \le t \le S_j^q + p_j^q \right\}$$

and the size of the engaged $k$-th resource the $q$-th project:

$$r_k(S^q,t) = \sum_{j \in A(S^q,t)} r_{jk}$$

where

$r_{jk}$ – is the size of the engaged $k$-th resource the $j$-th activity.

The portfolio of projects can be seen as a single project with inconsistent graph of activities. The addition of this new project portfolio can cause conflicts in the access to resources. We must do this so that these conflicts do not arise. For this purpose, we will create appropriate schedule of operations. It is a very complex task. To simplify this, we specify strategic resources from which you can start scheduling.

The method for finding strategic resources proceeds in the following steps:

**Step 1**: Identify a strategic resource for solving the problem of goal programming:

$$\begin{aligned}
& min \sum_k y_{kt}^+ \\
& s.t. \\
& S_{n+1}^q \le C_{min} \qquad \forall q \in Q \\
& S_j^q - S_i^q \ge p_i^q \qquad \forall i \in V^q - \{n+1\}, \forall j \in V^q - \{0\}, \forall q \cup q+1 \\
& \sum_q r_k(S^q,t) - y_{kt}^+ + y_{kt}^- = R_k^\rho \qquad \forall k \in \Re^\rho \ \forall t > 0 \\
& y_{kt}^+, y_{kt}^- \ge 0 \qquad \forall k \in \Re^\rho
\end{aligned}$$

where :

$y^{+}_{kt}$ – demand over the availability of resource $k$ at time $t$
$y^{-}_{kt}$ – demand below the availability of resource $k$ at time $t$
$C_{min}$ – declared time of completion of all projects

If objective function is equal 0 we have feasible schedule. Resources for which

$$\min_{k} \sum_{t} y^{-}_{kt}$$

is strategic resource.

**Step 2**: Add a new project activities, starting from those that use a strategic resource.
**Step 3**: By returning to Step 1, determine a new strategic resource.

In the first step, by solving the problem of goal programming, we are finding strategic resource. For this resource, we create a schedule by adding the activities of new project (Step 2). Adding them may cause that the new resource will become strategic, therefore in Step 3, we repeat the procedure. The proposed heuristics should significantly reduce the complexity resolved during scheduling issues.

## 4. Numerical example

In order to illustrate the proposed algorithm is considered a portfolio of projects considered in the literature (Hanh Quang Le 2008). We look at the portfolio as a single project activities inconsistent graph as shown in Figure 1 Summary of actions making up the designs shown in Table 1.

Table 1

Activities of the project portfolio

| Activity *j* | $p_j$ | *Pred(j)* | Resources |
|---|---|---|---|
| *Project P1* | | | |
| *P1-1* | 3,33 | | 2A, C |
| *P1-2* | 4 | | 2A, B, C |
| *P1-3* | 5 | *P1-1, P1-1* | A, C |
| *Project P2* | | | |
| *P2-1* | 5 | | A,B,C |
| *P2-2* | 4 | *P2-1* | 2A, |
| *P2-3* | 5 | *P2-1* | A,2B,C |
| *P2-4* | 4 | *P2-2, P2-3* | 2A,C |

Source: Hanh Quang Le: *Resource-Constrained Multi-Project Scheduling with Resource Moving Time for Construction Projects in Vietnam*, p. 103.

Availability of resources used (A, B, C) is shown in in Table 2.

Table 2

Availability of resources

| Resource | A | B | C |
|---|---|---|---|
| Availability | 4 | 2 | 2 |

Source: Ibid.

Graph of activities of the project portfolio is shown in Figure 1.

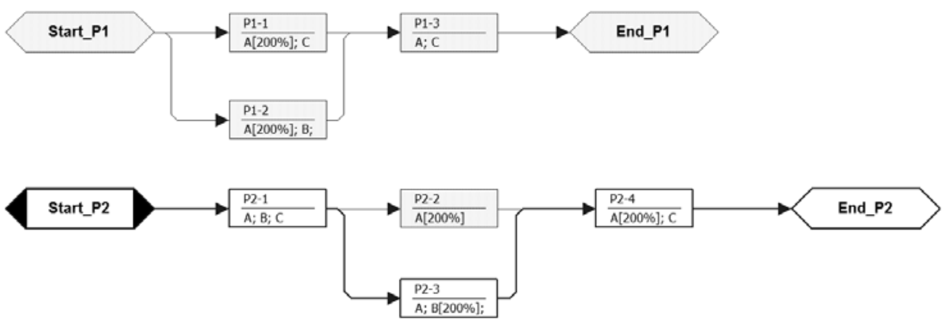


Figure 1. Graph of activities of the project portfolio

So defined problem has balanced resources to the implementation of the projects within the assumed time for schedules:

$$S^1 = (1, 1, 5)$$

$$S^2 = (5, 10, 10, 15)$$

Which are presented on Figure 2.

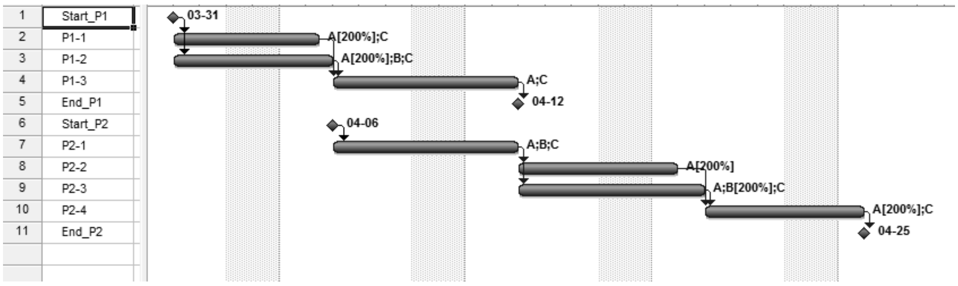


Figure 2. Project portfolio Gantt chart

Start a new project, will need to plan new activities that will be using the same resources. Considered a portfolio of projects, complemented by P3 project, which consists of the following activities: P3-1, P3-2 and P3-3 shown in Table 3. Graph of supplemented activities for this portfolio is shown in Figure 3.

Table 3

Activities of additional project

| Activity *j* | $p_j$ | *Pred(j)* | Resources |
|---|---|---|---|
| *Project P3* | | | |
| *P3-1* | 5 | | 2A, C |
| *P3-2* | 4 | | 2A, B, C |
| *P3-3* | 5 | *P2-1, P2-1* | A, C |

If we want to add this new project to the portfolio, it is necessary to identify a strategic resource in old one.

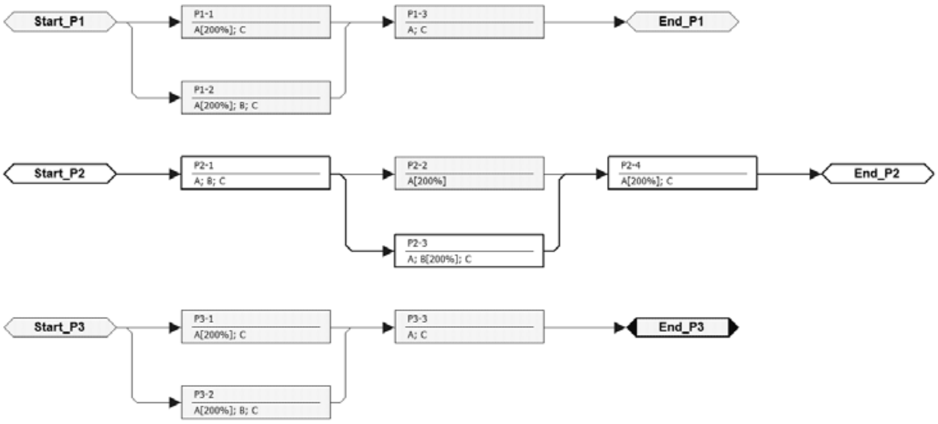


Figure 3. Supplemented project portfolio activities graph

**Step 1:** Implementation of the proposed method for the example shown, gives us, values of balancing variables are presented in Table 4 to 6 for all resources.

Table 4

Values of balancing variables for resource A

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y^+_{kt}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y^-_{kt}$ | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 4 | 4 |

Table 5

Values of balancing variables for resource B

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y^+_{kt}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y^-_{kt}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 |

Table 6

Values of balancing variables for resource C

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y^+_{kt}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y^-_{kt}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 |

It gives C as strategic resource. (Sum is equal to 11)

**Step 2**: We add activities of project P3 which use resource C. That is activity P3-1. First moment when we have enough is day 10, but other resources are not available (A) . We can start this activity in day 15.

**Step 3**: By returning to Step 1, we must determine a new strategic resource.

By continuing the procedure, we obtain a schedule for the project *P-3* as:

$S^3 = (14, 19, 23)$

which is presented as Gantt chart on Figure 4.

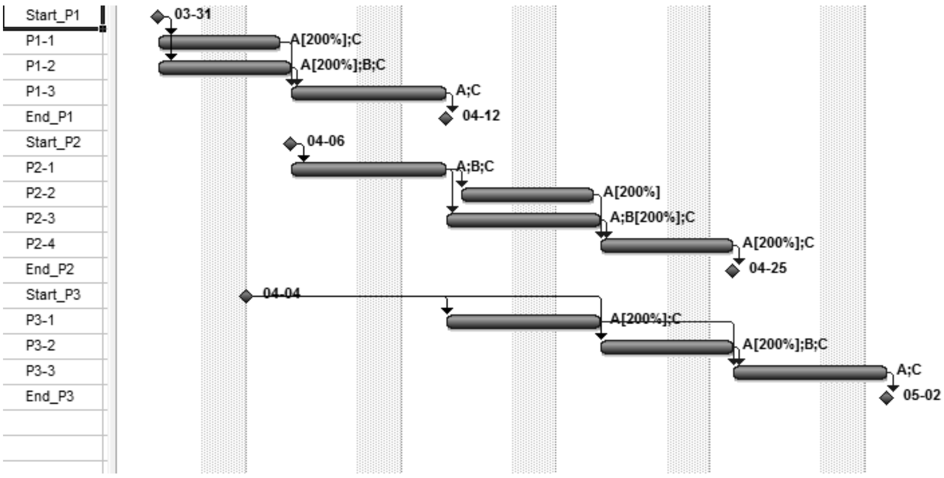


Figure 4. Gantt chart for modified portfolio

## Conclusions

The paper presents a heuristic algorithm scheduling support project portfolios. It uses goal programming method, in order to identify critical resources within the meaning of E. Goldratt Theory of Constraints. Operation of heuristics is shown in a simple example. Its practical effectiveness can only be fully assessed until the computer implementation. It is planned in AIMMS environment.

## References

A Guide to the Project Management Book of Knowledge (PMBOK 2004). 3rd Ed. Newtown Square, PA: Project Management Institute.

Artigues C., Michelon P., Reusser S. (2003), *Insertion techniques for static and dynamic resource-constrained project scheduling.* "European Journal of Operational Research", 1;149(2).

Ayala M., Artigues C. (2010), *On integer linear programming formulations for the resource-constrained modulo scheduling problem.* Technical Report 10393, LAAS-CNRS, Toulouse.

Brucker P., Drexl A., Mohring R., Neumann K., Pesch E. (1999), *Resource-constrained project scheduling: Notation, classification, models, and methods*. "European Journal Of Operational Research" No. 1.

Goldratt E.M. (1997), *Critical Chain.* Great Barrington, MA: The North River Press.

Hanh Quang Le (2008), *Resource-Constrained Multi-Project Scheduling with Resource Moving Time for Construction Projects in Vietnam.* First International Conference on Construction in Developing Countries (ICCIDC–I).

Jarboui B., Damak N., Siarry P., Rebai A. (2008), *A combinatorial particle swarm optimization for solving multi-mode resource-constrained project scheduling problems.* "Applied Mathematics and Computation" No. 15, 195(1).

Merkle D., Middendorf M., Schmeck H. (2002), *Ant colony optimization for resource-constrained project scheduling*. "IEEE Transactions On Evolutionary Computation" Vol 6(4), 333-343.

Mika M., Waligora G., Weglarz J. (2005), *Simulated annealing and tabu search for multi-mode resource-constrained project scheduling with positive discounted cash flows and different payment models*. "European Journal of Operational Research", 1;164(3), 639-668.

Neumann K., Zimmermann J. (2000), *Procedures for resource leveling and net present value problems in project scheduling with general temporal and resource constraints.* "European Journal of Operational Research", 1;127(2), 425-443.

Steyn H. (2002), *Project management applications of the theory of constraints beyond critical chain scheduling.* „International Journal of Project Management”, 20(1), 75-80.

Targiel K. (2012), *The method of dynamic scheduling of portfolios of projects with constrained resources,* in: *Modelowanie Preferencji a Ryzyko’12.* t. 97, Ed. T. Trzaskalik, Studia Ekonomiczne, Zeszyty Naukowe Wydziałowe. Wydawnictwo Uniwersytetu Ekonomicznego w Katowicach, Katowice, 315-324 (in Polish).

Tormos P., Lova A. (2003), *An efficient multi-pass heuristic for project scheduling with constrained resources.* “International Journal of Production Research”, 41(5), 1071-1086.