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A SUBJECTIVE APPROACH IN RISK MODELING USING SIMULATION TECHNIQUES

Introduction

Quantitative risk assessment basing on simulation techniques mainly concentrate on historical risk information. Financial companies have been used so far, to perform comprehensive historical data collection concerning key risks. Financial companies' information technology provides not only complex historical information, but gets the information with accurate frequency as well. Financial risk information is assessed by financial markets themselves (by proper financial institutions), thereafter shared with market participants, whereas operational and credit risk information has to be collected by financial companies on their own. Equipped with proper risk information, financial companies are able to model their behavior in volatile environment finding their actual risk exposure. Non-financial companies are even more uncertain about their future, though putting risk modeling aside. The main reason of inclining the modeling, may be historical data availability. Of course, a data collection, similar to financial companies' systems, is possible to be introduced in non-financial companies. Though bringing such the data collection into a company can be an expensive process, especially for small and medium enterprises (SME). Not knowing exact future benefits, non-financial companies, can likely incline such systems. Historical data problems should not prejudge risk modeling resignation. SMEs can introduce risk modeling approach basing on subjective assumptions involving both risks' distributions and interdependencies. Having built a valid model concerning given financial situation, one can model risk basing on special – subjectively chosen – distributions. Triangular and beta distributions work especially great when an expert opinion is the only data source (Vose, 2008). In this study, risk adjusted performance analysis, using simulation techniques with subjective assumptions, is presented. An investment projection model is used, to present both opportunities arising from making subjective assumptions and threatens arising from not taking interdependencies into account. Frequency function is presented as an easy to interpret alternative to probability density functions and cumulative probability distribution functions in parallel. Frequency based approach is considered, when subjective assumptions arise from an expert opinion, who's statistical knowledge remains rather poor.

1. Subjective assessment better than scenarios?

Simulation techniques like Monte Carlo Simulation (MCS) or Latin Hypercube Simulation (LHS) can be considered as an evolution of classic scenario analysis. In fact, there are hundreds of thousands scenarios being randomly generated during both MCS or LHS as well. Every scenario is a set of random values of risk factors obtained compliantly to assumed probability distributions. Scenarios are processed iteratively in relevant financial model in order to gain risk variables' probability distributions. The main idea of simulation techniques is to analyze as much scenarios as possible, finding every logical situation likely to happen (Vose, 2008). In traditional scenario analysis, in turn, only a few scenarios, with subjectively attributed probability, were generated mostly showing an enterprise: fully exposed to downside risk, not exposed to risk, fully exposed to upside risk. Simulation techniques give an opportunity to consider lots of combinations when some risks gets their upside values whereas others – their downside values. Both mentioned methods obtain, in fact, a risk variable's probability distribution, but the comprehensiveness votes for simulation (Fig. 1).

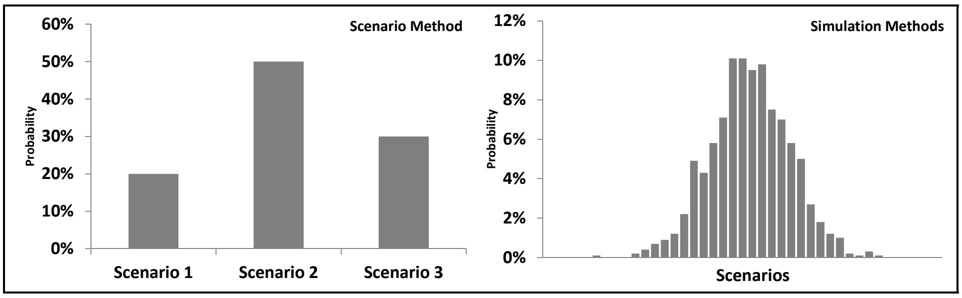

Fig. 1. The nature of the outcome – classic scenario analysis versus simulation techniques

Historical data availability poses a major issue while assuming risk factor's probability distributions. Historical values are considered as objective source of information. Vose (2008), Chapman (2006) agree on number of various situations, when objective way might have been considered as a serious hurdle:
– the data has simply never been collected before,
– the data is too expensive to obtain,
– past data is no longer relevant,
– the data is sparse requiring expert opinion "to fill in the holes",
– the area being modeled is new.

Recapitulating above, risk factors' distributions may be attributed in: objective way, quasi-objective way or subjective way, depending on both historical data availability and adequacy as well. The non-historical descent of risk factor's probability distribution doesn't cross simulation techniques out. Expert opinion can be the source of right distribution, even if the possessed information consists only of the risk factor's extreme values. There exist a number of theoretical distributions being suitable to summarize, more or less detailed information gained from experts knowing best the nature and the behavior of a particular risk factor.

2. Subjective assessment using triangular distributions

Using triangular distributions for simulation reasons doesn't seem to be particularly challenging. Simulation techniques require convenient inverse cumulative distribution functions (G(α)), enabling the right sampling process*. Any professional risk software** provides proper triangular distribution functions, whereas popular spreadsheets don't. The best known, Microsoft Excel, provides object oriented programming using Visual Basic for Applications (VBA). Preparing suitable VBA functions could have been quite usable solution involving low budget, making subjective assessment with triangular distributions possible.

The common approach is using simple triangular distributions described only by their extreme values, with an assessment considering which of them has the highest probability of occurrence (Kaczmarzyk, Zieliński, 2010).

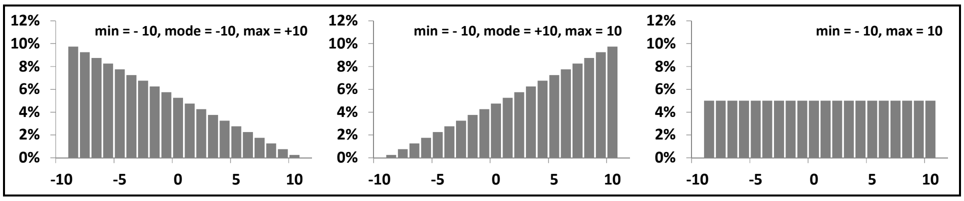

Fig. 2. Simple triangular distributions and uniform distribution

Simple triangular distributions seem to be particularly suitable solution, when the only thing the analyst can do is describe the range of possible risk factor's values. Depending on the highest expected probability, analyst should choose left or right skewness of the simple triangular distribution. Finding the highest expected probability hard to describe, an analyst may use uniform distribution, while assuming the same probability for the expected range of values (Fig. 2).

* Sampling process – generating random numbers due to assessed probability distributions, consist of two stages. Stage 1 – generating uniform random numbers from range (0,1) (generating probability in fact). Stage 2 – transforming uniformly distributed numbers into desired probability distributions using inverse cumulative distribution functions.

** Fe. ModelRisk, Palisade Risk, Crystal Ball.

The more complex approach involves universal triangular distributions (Vose, 2008), enabling analysts to assume the value with the highest probability somewhere between expected extreme values (with the lowest probability).

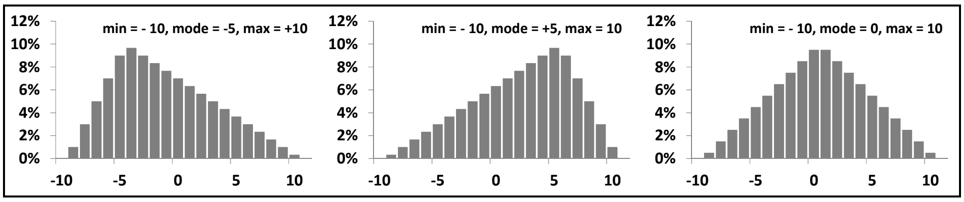

Fig. 3. Universal triangular distribution with same extreme values and different expected values

One has to magnify the nature of universal triangular distribution, which boils down to the fact, that the values nearly the extreme values have equal probability, only when the most expected value lays exactly in the middle of the expected range (Fig. 3).

In terms of financial categories, one is able to provide lots of examples when triangular distributions are a right solution. Let's assume a company considering an investment project and its investment expenditures. Typically, the company lacks of historical information being forced to simulate risk using a subjective assumption. Choosing the triangular distributions, is expected to provide the minimum, maximum and most expected value of the investment expenditures. Defining key distribution values, may necessitate some consultations with experts. In other words, one has to consult if there is a chance to decrease costs and if there is a risk of their increase. Involving brainstorming and other creative thinking techniques may provide desired information in much more effective way.

Triangular distributions are very easy to interpret, even for persons lacking of statistical experience. In other words, risk identification, even on the lowest level of an organization, can be effective, basing on information possessed from serial workers. Let's assume a company holding production lines, which doesn't have information on their actual reliability. Employees responsible for particular production line are likely to have such an information acquired automatically straight from the production process. Obviously, an immediate information won't be much more detailed than required by triangular distributions themselves. Looking for lots of details, company has to introduce special data acquisition process, involving employees from adequate level of organizational structure. Such solution could provide the most accurate probability distribution fit.

Finally, the main constraint of using triangular distributions is linear relation between risk factor's values inside expected range and their probability of occurrence. The other significant constraint of triangular distributions is inability to differ the expected values' probabilities for different risk factor's having the same range of volatility (with both different and same expected values). Being conscious the real risk factor's nature, one may use beta probability distributions instead, able to reflect the non-linear relation and to differ the probability of the expected value.

3. Subjective assessment using beta distributions

Every theoretical distribution could be used in making the subjective assumptions. Lots of them have complicated parameters which cross statistically inexperienced experts out when it comes to simply draw the risk. Even normal distribution might cause some difficulties while realizing its true volatility range doesn't cover straight with the standard deviation. Some of the theoretical distributions are easy to parameterize instead like triangular ones. One of the most useful distributions is beta which taps the same parameters as the universal triangular distributions mentioned above. Unlike triangular distributions, beta ones are able not only to position the expected value but to set the expectancy strength as well.

Beta general distributions work with a specific set of parameters: α, β (both responsible for shape), minimum and maximum. Beta general gets symmetrical shape while shape parameters are equal ($\alpha = \beta$), in the other cases the distribution will remain asymmetrical. Beta general is capable of forming lots of shapes, making the distribution highly universal, especially when the expert opinion is urgent to take shape (Fig. 4).

Setting the beta general distribution's parameters seems to be quite comfortable when having in mind symmetrical shape. The higher α and β, assuming $\alpha = \beta$, the wider the distribution's volatility range. Statistical experience is highly recommended when asymmetrical shape is the key, when describing particular risk factors. Quite useful may be David Vose's (2008) approach for asymmetrical beta distributions, leading towards parameters simplification including extreme values (min and max), mode and shape. Vose's algorithm needs an addition for symmetrical cases as follows (1) and can be a perfect basis for urgent changes in existing Excel's Beta. (e.g. by creating new function on the basis of the built-in-excel one).

$$\alpha = \frac{(\mu - min)(2mode - min - max)}{(mode - \mu)(max - min)}$$

$$\beta = \frac{\alpha(max - \mu)}{(\mu - min)}$$ if $max - mode \neq mode - min$ (asymmetrical distributions) (1)

where $\mu = \frac{min + shape \cdot mode + max}{shape + 2}$

$$\alpha = \beta = \frac{shape}{2} + 1$$ if $max - mode = mode - min$ (symmetrical distributions)

The shape's parameter determines its kurtosis (the distribution's flattening). The higher the shape's value the lower the kurtosis and the distribution's volatility as well. The recommended solution, when it comes to eliciting the risk distribution from an expert opinion, is to share a suitable legend, presenting beta distributions with different parameters (Fig. 4).

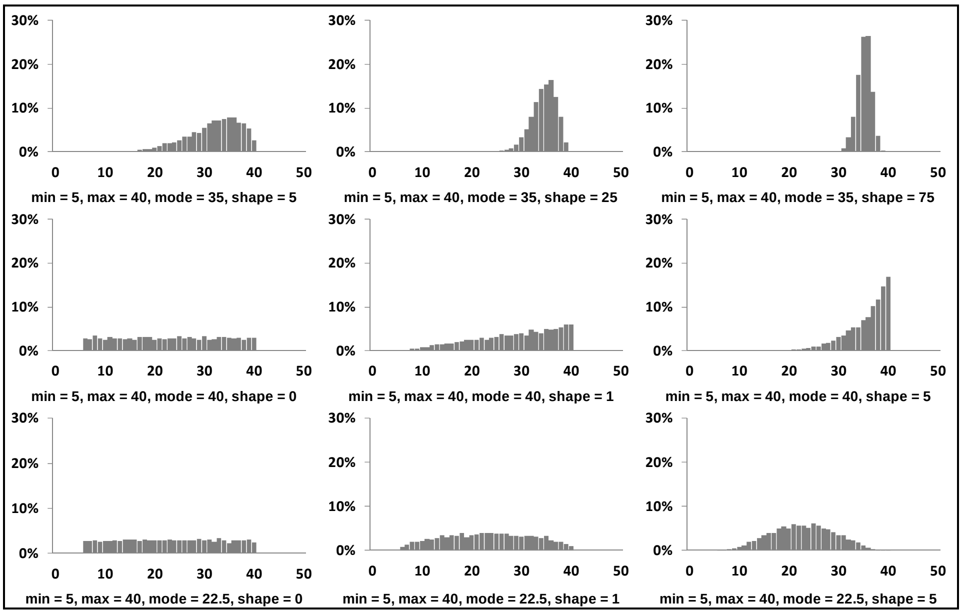

Fig. 4. Beta distributions with the simplified set of parameters

4. Interdependencies' dilemma

There is a huge problem with reflecting interdependencies between risks in risk analysis. Likewise assuming the risk distributions subjectively, the interdependencies may be elicited from an expert opinion, while historical data is inaccessible or unavailable. Having in mind the interdependencies seems to be essential. The previously recalled investment project, can be a suitable example once again. Presumably there are two different goods' production lines to be launched. Expected revenues from the production lines won't be independent from each other. As a consequence of diversification level, the revenues changes may exhibit positive or negative correlation as well. Furthermore the omission of interdependencies is going to be, in fact, an assumption reflecting particular level of diversification.

The relevant method of reflecting the interdependencies should work with most common correlation measure such as the Pearson's coefficient (or its conversion as the coefficient of determination*). One of the simplest and the most universal methods, is the Cholesky's decomposition providing so called normal copula**. Either bivariate (Jäckel, 2002) or multivariate interdependencies' (Cherubini, Luciano, Vecchiato, 2004) problem is easy to be solved using the decomposition. Turning towards the mechanism, the Cholesky's decomposition converts standardized bivariate or multivariate normal distribution with independent vectors into a relevant distribution with dependent ones.

Choosing the right copula is the another significant challenge in the subjective risk modeling. The normal copula achieved with the Cholesky's method doesn't reflect interdependencies nature properly in some circumstances, especially when it comes to the financial companies activity and tail dependence between distributions (Melchiori, 2003; Kole, Koedijk, Verbeek, 2007). Knowing best the right type of the copula, forces the copula fitting process which absolutely requires the historical data.

5. Subjective assumptions and correct charts

Experienced statisticians or financials have embedded-by-experience ability to understand probability density function (PDF) and cumulative distribution function (CDF). The experts whose statistics remains rather poor, may find useful a frequency distribution function (FDF), especially when denominated in percentage points. One supposes the FDF to be clear for nearly anyone, while using "percent from population within range" in fact. The experts are going to fully understand and properly choose, when presented the possible FDF's examples instead of the PDF's or CDF's.

* Using the coefficient of determination maybe actually quite comfortable solution while making the subjective assumptions. Stating the value of the determination's coefficient is much clearer The only thing one has to state is the part of the risk factor's changes which contribute to changes of the another.

** A copula is a particular kind of interdependency between probabilities of the risk factors. The normal copulas form characteristic elliptical shapes. Looking for the best fitted copula is looking for the right shape in fact.

6. Subjective assumptions in practice

A simple profitability model is presented for illustrating the subjective assumptions idea (Fig. 5). The model calculates return on equity (ROE) within one year horizon for two production lines financed partially with debt. A risk analysis is conducted with taking into account market risk appearing in the products' prices. The MCS sampling is used with the Cholesky's decomposition for reflecting possible interdependencies between the prices.

Product A

Price per Unit	200,00 zł
Quantity (Units)	2000
Variable Cost per Unit	140,00 zł

Product B

Price per Unit	500,00 zł
Quantity (Units)	3000
Variable Cost per Unit	215,00 zł

Other details

Own Capital	700 000,00 zł
Debt Capital	900 000,00 zł
Fixed costs	400 000,00 zł
Interest Rate	9%
Tax Rate	19%

Correlation Coefficient
0,8

Projected Income

Sales	1 900 000,00 zł
Variable Costs	925 000,00 zł
Fixed Costs	400 000,00 zł
EBIT	**575 000,00 zł**
Interests	81 000,00 zł
EBT	**494 000,00 zł**
Taxes	93 860,00 zł
EAT	**400 140,00 zł**

ROE	**57,16%**

Fig. 5. Simple profitability model, considering return on equity

Let's assume the company is expecting the price for the product A can change within a range of 150,00 zł to 290,00 zł with the most expected price's level at 200,00 zł. Relevantly, product B can change within a range of 300,00 zł to 550,00 zł with the most expected price's level at 500,00 zł. The MCS brings the ROE's distribution, which differs seriously when changing correlation strength between the prices. Checking the ROE at risk with 10% level of significance, one gets following results for different level of the correlation coefficient (ρ):

1. For ρ = +0,8 (the prices behave rather similarly), the ROE is going to be higher than 7,6% with 90% probability (Fig. 6).
2. For ρ = 0,0 (the prices behave independently), the ROE is going to be higher than 14,2% with 90% probability (Fig. 7).
3. For ρ = -0,8 (the prices behave rather contrariwise), the ROE is going to be higher than 22,1% with 90% probability (Fig. 8).

Making traditional scenarios wouldn't have brought the ROE's related information with probability level in such detailed way. Calculating the ROE with simulation techniques brings more comprehensive image of risk when managing finance in a company. Even the triangular distributions enable an analyst to simply consider as much scenarios as possible.

The interdependencies are also crucial. Supposing an analyst is not going to take interdependencies into account while there is a strong positive (or negative) correlation between risks. Missing the interdependencies is going to provide underestimated (or overestimated) risk (e.g. Fig. 6, Fig. 7, Fig. 8).

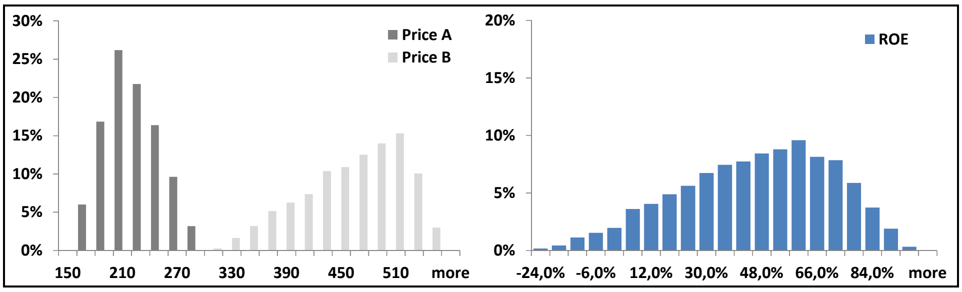

Fig. 6. Example 1, Price A: Triangular min = 150, mode = 200, max = 290; Price B: Triangular min = 300, mode = 500, max = 550; ρ = + 0,8

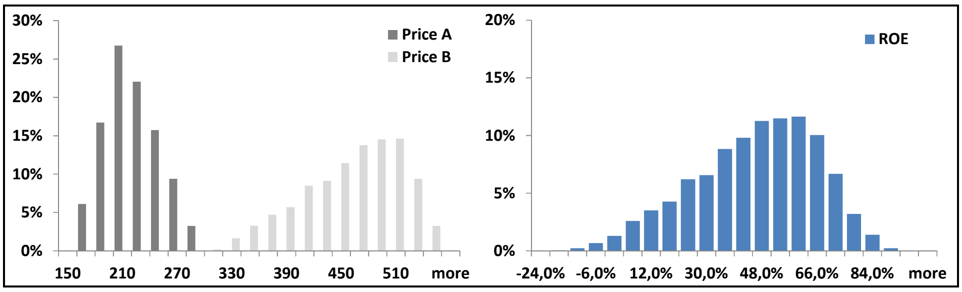

Fig. 7. Example 2, Price A: Triangular min = 150, mode = 200, max = 290; Price B: Triangular min = 300, mode = 500, max = 550; ρ = 0,0

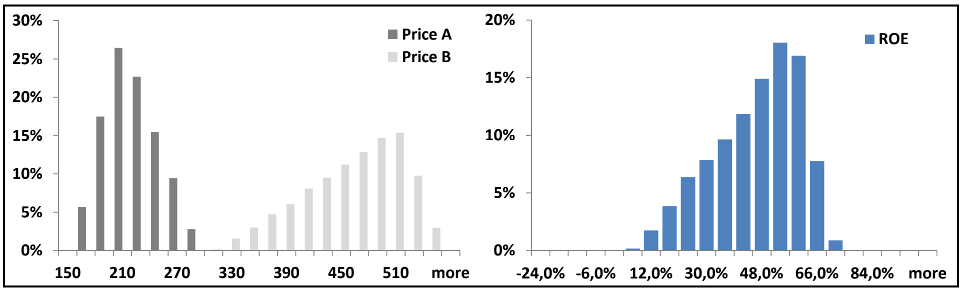

Fig. 8. Example 3, Price A: Triangular min = 150, mode = 200, max = 290; Price B: Triangular min = 300, mode = 500, max = 550; ρ = – 0,8

Alternatively, as has been stated so far, one is able to make subjective assumptions using beta distributions. In order to compare the beta distributions with the triangular ones, the same extreme values and modes were set. The main advantage benefited from the beta distribution is the ability to easily change the shape. In following examples the shape's parameters were replaced, maintaining the others (Fig. 9, Fig. 10, Fig. 11).

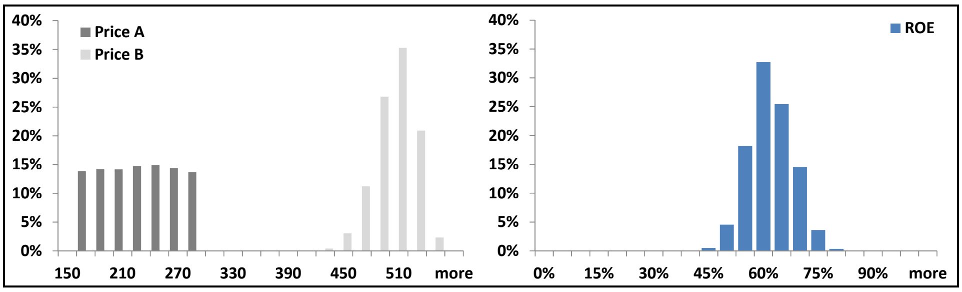

Fig. 9. Example 4, Price A: Beta min = 150, mode = 200, max = 290, shape = 0; Price B: Beta min = 300, mode = 500, max = 550, shape = 20; ρ = -0,8

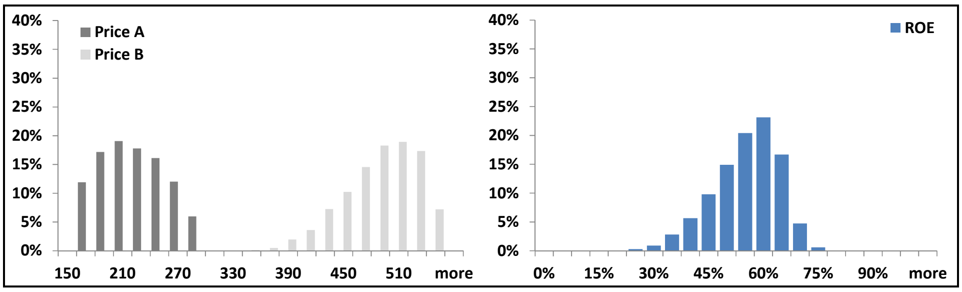

Fig. 10. Example 4, Price A: Beta min = 150, mode = 200, max = 290, shape = 1; Price B: Beta min = 300, mode = 500, max = 550, shape = 5; ρ = -0,8

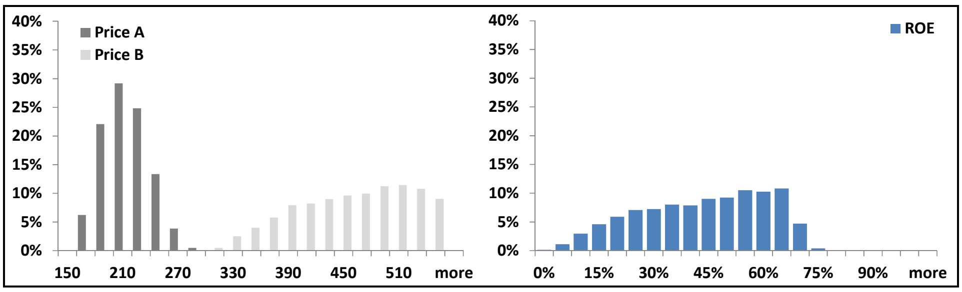

Fig. 11. Example 4, Price A: Beta min = 150, mode = 200, max = 290, shape = 5; Price B: Beta min = 300, mode = 500, max = 550, shape = 1; ρ = -0,8

Furthermore, the FDFs endorse their ability to present the probability as simple as possible. The CDFs with cumulative information could heavily blur probability images. The presented model assumes Product B to bring much higher profit margin than Product A. One is able to possess the same information, while looking at the prices' and the ROE's FDFs – the ROE significantly tends to maintain the shape of much more profitable Product B.

Summary

Whenever risk analysis is important, one has to consider using simulation techniques. Having historical data in hand may be both useful and dangerous as well. Even historical information has to be transformed in some way to truly reflect future nature of an economical process. It is suggested that companies shouldn't cross simulation, when the subjective way, is the only way on the horizon. Even subjectively chosen distribution can bring much more detailed picture of the company's risk. Triangular and beta distributions seem to be really helpful when it comes to picture risk factors without historical data. Empowering the analysis with subjectively chosen distributions with interdependencies' assumptions eliminates some illogical scenarios from simulation process and can't be put aside. The only hurdle is the convenient software. Using spreadsheets is suggested but involves two approaches. First, one can possess license for using some professional add-ons (like @Risk, Crystal Ball etc.). Second, one may develop a model oneself. First approach is rather expensive, whereas second necessitates proper IT experience.

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PODEJŚCIE SUBIEKTYWNE W MODELOWANIU RYZYKA Z WYKORZYSTANIEM TECHNIK SYMULACYJNYCH

Streszczenie

Zastosowanie technik symulacyjnych powinno być brane pod uwagę zawsze w sytuacji, gdy konieczne jest przeprowadzenie analizy ryzyka. Dostępność danych historycznych nie powinna być ostatecznym kryterium wyboru technik symulacyjnych. Subiektywny dobór rozkładów czynników ryzyka oraz współzależności pomiędzy nimi

może stanowić wyjątkowo atrakcyjne i skuteczne rozwiązanie. Zaletą wykorzystania technik symulacyjnych jest możliwość rozważenia ogromnej liczby wariantów, szczególnie w zestawieniu z tradycyjną metodą scenariuszy. Uwzględnienie współzależności eliminuje ponadto nierealne scenariusze. Część teoretycznych rozkładów prawdopodobieństwa w szczególny sposób ułatwia subiektywne założenia w analizie ryzyka, w sytuacji gdy opinia eksperta jest jedynym źródłem informacji o ryzyku. Prezentowany jest pogląd, iż przedsiębiorstwa powinny rozważyć wykorzystanie technik symulacyjnych w procesie zarządzania ryzykiem, podobnie jak czynią to instytucje finansowe.