Bogumiła Krzeszowska
Department of Operations Research
University of Economics in Katowice

PROJECT PORTFOLIO SCHEDULING AS A MULTIPLE-CRITERIA DECISION MAKING PROBLEM

Introduction

One of the most important phase of project management is planning. During this phase tasks are identified and scheduled. Schedule consists of a list of a project`s activities with planned start and finish dates. That is why scheduling is a critical issue in project management. "Scheduling is to forecast the processing of work by assigning resources to tasks and fixing their start times. […] the different components of a scheduling problem are tasks, the potential constraints, the resources and the objectives…" (Carlier, Chretienne 1988). Scheduling concerns the allocation of limited resources to tasks over time. It is a decision-making process that has a goal – the optimization of one or more objectives (Pinedo 1995).

In project oriented companies many projects are managed at the same time. Projects are organized in portfolios. a portfolio is a collection of projects or programs grouped together to facilitate effective management to meet strategic business objectives. Those projects use resources from one set and influence the economic aspects of all portfolio.

The main project scheduling techniques are CPM and PERT. In the CPM method the longest path of planned activities to the end of the project is calculated. It delivers information about the shortest time of project realization. The Program Evaluation and Review Techniques (PERT) is a method to analyze involved tasks in completing a given project, especially the time needed to complete each task. Those methods deliver schedules with optimal project finish time. In real-life applications the schedule should optimize not only project finish time but also resource usage and cash flows.

The purpose of this paper is to present project portfolio scheduling problem as a multiple criteria decision making problem and to propose a methodology for solving this problem. An example of it project portfolio will be presented as an illustration. A schedule for portfolio of ERP deployment projects will be described. Projects have very similar structures and can be presented on the same network.

The paper begins with a rerview of literature and problem statement. Then, the methodology for project portfolio scheduling problem is described. Methodology consists of three parts: mathematical model building, non-dominated solutions identification and final solution choosing. Paper finishes with conclusions and ideas for future research.

1. Optimization in project scheduling problem – a literature review

We can assume, that there are three main factors influence project schedule. Those are: time, resources and economical factors, e.g. costs or project NPV. A time can be an optimization criterion (we aim to finish the project as fast as possible), but in many cases it is also constrained. Each activity requires some types and amount of resources. Resources are constrained in a period of time. We can also try to optimize the usage of the resources. Each project needs costs for its realization, but it is also an investment, so it should maximize the NPV.

When we take into consideration all the factors mentioned above we can build various mathematical models for project scheduling problem. Each mathematical model for project scheduling problem should include the precedence |relationship constraints and information about the extent of variables. We can also vary problems in terms of number of objectives, so there are: one objective project scheduling problems and multiple criteria project scheduling problems.

In research on project scheduling problem one objective models are the most popular. Multiple criteria optimization in this field is not frequently undertaken. Also project portfolio scheduling problem is rarely presented in the literature.

A resource constrained multiple criteria project scheduling problem was presented by Viana and de Sousa (2000). A mathematical model in which: project completion time is minimized, project delay is minimized and disorders in resource usage are minimized was proposed.
A binary variables x_{ijt} are used in the model:

$$x_{ijt} = \begin{cases} 1 & \text{when operation } j \text{ of an activity } i \text{ is finished in time} \\ 0 & t \text{ otherwise} \end{cases}$$

Lova, Maroto and Tormos (2000) presented a paper in which a multiple criteria project scheduling problem is solved in two phases. In the first phase an optimal, in terms of time criterion, schedule is prepared. In the second phase, the schedule obtained in the first phase is improved in terms of resources usage.

Leu and Yang (1999) presented multiple criteria resource constrained project scheduling problem with time, cost and resource usage optimization.

Hapke, Jaszkiewicz and Słowiński (1998) also described the multiple criteria project scheduling problem in their paper. They considered three criteria: project delay minimization, project cost minimization and resource usage optimization.

All papers presented above consider one project scheduling problem. In the case of project portfolio scheduling problem, one objective resource constrained model is considered by few authors. Goncalves, Mendes and Resende (2004) described a resource constrained project portfolio scheduling problem with time optimization. Chiu and Tai (2002) presented resource constrained project portfolio scheduling problem. They proposed a criterion function in which they join NPV and penalty for projects delays.

2. Problem statement

A company from the Silesia region deploys an ERP system. There are about 30 deployment projects managed simultaneously by 8 teams. For each project we can define some activities, which are the same for all projects in the portfolio. a single project consists of 37 activities (those activities have been described in the company`s project guide). Project contains of 11 main phases.

Company signs a contract with customer, which includes information about the main issues of the project. in this contract, a due date and penalty for project delay is defined. a payment for the project is also defined in the contract as well. Payment is split into 11 parts. At the end of each of the 11 phases customer pays 1/11 of the total amount.

There are about 10-12 employees in the project team. in case of lack of resources, project manager can hire the resources from other teams. External resources are extra paid. the availability of resources can be different in each period of time. Team manager cannot use more resources than it is available in the company. One team leads about 3-4 projects simultaneously. Projects did not started at the same time. One project can start and other one can be at the final phase.

Each project portfolio manager wants to maximize its profit. That is why project should be finished on time, external resources should not be used and the project portfolio schedule should maximize its NPV.

2.1 Current situation

Currently, projects in the portfolio are managed independently by a project portfolio manager. Scheduling is based on Gantt chart and CPM method and schedules are prepared only for one project. The schedule is prepared at the beginning of the project and is presented to the customer as a baseline to the contract regulations. Analysis of resources availability are not prepared before a contract is signed. If new project appears in the portfolio it is scheduled independently, without analyzing its influence the other projects in the portfolio. Resources availability is not controlled during the scheduling phase. Sometimes company hires external resources to meet the deadlines.

3. Methodology for project portfolio scheduling problem solving

Analysis of the current situation in the company is a base for methodology preparation. Project portfolio schedule should be prepared in each case, when some changes occurs, e.g. new project introduction, change in the resources availability, change in project scope, contract renegotiation. Each of those situations influence the project portfolio schedule. That is why a methodology and tool, which will prepare a schedule in the short period of time is necessary.

Project portfolio scheduling problem described above consists of many factors, like contract statements, resources availability and goals for project portfolio manager. to build a methodology to solve this problem some general assumptions need to be prepared.

The following assumptions were made:

- Projects in portfolio shouldn`t be delayed or the sum of penalty for project delay should be minimized. the penalty is charged for each period of time, but the total amount cannot be higher than the maximum penalty defined in the contract. a grace period is foreseen, in which the penalty is not charged.
- Project manager should use internal resources to manage the project portfolio and minimize the costs of the external resources usage. the resource usage cannot exceed the availability of resources in the company.
- The schedule should maximize the sum of NPV of all projects in portfolio.
- Only renewable resources are taken into consideration. We assume that the amount of nonrenewable resources needed for project execution is constant and is not limited for the period of time, but for the project. That is why we do not need to consider them in the model. if we will not have the relevant amount of non-renewable resources the project cannot be completed.

Single project structure was analyzed and described on AON (Activity on Node network). Activity on Node network was used (Figure 1), because various types of precedence relationships between project activities occurs. Activity on Arc network enables to use only finish-to-start precedence relationships. Milestones was identified and marked on the chart. In each milestone, company receives some amount of money for completed tasks.

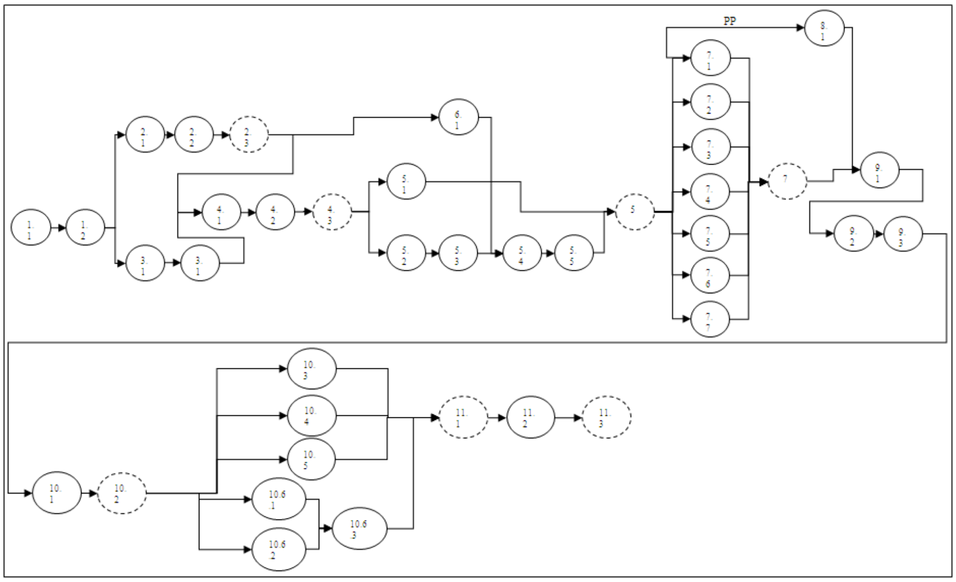

Figure 1. Activity on Node network for an ERP deployment project

Methodology for project portfolio scheduling problem proposed in this paper consists of the following steps:

1. Multiple-criteria mathematical model building.
2. Finding non-dominated solutions.
3. Choosing one solution from the Pareto set as the final solution.

At the beginning, a project portfolio scheduling problem is described as a multiple criteria decision making problem. Three main important factors are optimized: a time for project portfolio realization, resources used during each period of time and NPV for all projects in the portfolio.

In the next step an elitist evolutionary algorithm, to find non-dominated solutions, is used. The SPEA2 delivers a definite number of N non-dominated solutions.

In the last step, solutions obtained in the second step are analyzed and one schedule is chosen. The Light Beam Search (Jaszkiewicz, Słowiński 1999) procedure is used in this step.

3.1 Mathematical model

To build a mathematical model the following assumptions are made:

- there is a set of p similar projects $p=1,\dots,P$,
- a single project consists of $j=1,\dots, J$ activities,
- a time period is defined as $(t–1, t)$,
- project portfolio duration is constrained to T $(t=0,\dots, T)$,
- a single project has been presented on the AON network, so precedence relationships can be: finish-to-start, finish-to-finish, start-to-start, start-to-finish type,
- there are two resources types: internal ($R^w{}_{kt}$) and external ($R^z{}_{kt}$).
- availability of the resources can be different in each period of time.
- total availability of resource k in period t is $R^w{}_{kt} + R^z{}_{kt}$
- if the usage of internal resources is exceeded, activities use external resources,
- cash flows are generated at the end of the activity duration,
- projects in the portfolio can be on the various level of progress – one project can be in the 'project definition phase' and another on the 'programming phase',
- projects can depend on other projects – if there is a precedence relationship between activities of different projects,
- schedule is prepared every time, when there is a change in project portfolio, e.g. new project introduction, change in resources availability, etc.

Notation:

- x_{jpt} – decision variable,

$$x_{jpt} = \begin{cases} 1 & \text{when an activity } j \text{ of project } p \text{ last in period } t \\ 0 & \text{otherwise} \end{cases}$$

- d_{jp} – duration of activity j of project p,
- r_{jpk} – amount of renewable resource k required by activity j of project p,
- $R^w{}_{kt}$ – internal renewable resource k availability in time t,
- $R^z{}_{kt}$ – external renewable resource k availability in time t,
- F_{jp} – finish time of activity j of project p,
- S_{jp} – start time of activity j of project p,
- LF_{jp} – latest finish time of activity j of project p from the CPM method,
- Y_p – grace period in project p,
- Z_p – penalty for project p delay,
- V_p – penalty for overuse of internal renewable resource k,
- α – discount rate,
- cf_{jp} – net cash flow generated by activity j of project p,
- A_{jp} – set of predecessors i $(i=1,\dots, I)$ of activity j of project p.

Multiple criteria resource constrained project portfolio scheduling problem can be formulated as follows:

$$\sum_{p=1}^{P}[Z_p \times \max\{[[\sum_{j=1}^{J}\max\{0, F_{jp} - LF_{jp}\}] - [\sum_{i=1}^{I}\max\{0, F_{ip} - LF_{ip}\}]] - Y_p, 0] \rightarrow \min (i \in A_{jp}) \quad \text{(C1)}$$

$$\sum_{t=1}^{T}[\sum_{k=1}^{K}[\max\{\sum_{p=1}^{P}\sum_{j=1}^{J}(r_{jpk} \cdot x_{jpt}) - R_{kt}^{w}, 0\} \times V_k]] \rightarrow \min \quad \text{(C2)}$$

$$\sum_{p=1}^{P}\sum_{j=1}^{J} cf_{jp} \cdot e^{-\alpha F_{jp}} \rightarrow \max \quad \text{(C3)}$$

$$x_{jpt} = \{0,1\} \qquad (j=1,\ldots,J,\ p=1,\ldots,P,\ t=1,\ldots,T) \quad (1)$$

$$\sum_{t=1}^{T} x_{jpt} = d_{jp} \qquad (j=1,\ldots,J,\ p=1,\ldots,P) \quad (2)$$

$$F_{jp} = \max_{t=1,\ldots,T}\{(t \times x_{jpt})\} \qquad (j=1,\ldots,J,\ p=1,\ldots,P) \quad (3)$$

$$S_{jp} = \min_{\substack{t-1,\ldots,T \\ x_{jpt} \neq 0}}\{(t \times x_{jpt})\} - 1 \qquad (j=1,\ldots,J,\ p=1,\ldots,P) \quad (3')$$

$$F_{jp} = S_{jp} + d_{jp} \qquad (j=1,\ldots,J,\ p=1,\ldots,P) \quad (3'')$$

$$S_{jp} \geq F_{ip} \qquad (j=1,\ldots,J,\ p=1,\ldots,P,\ i \in A_{jp}) \quad (4')$$

$$\sum_{p=1}^{P}\sum_{j=1}^{J}(r_{jpk} \cdot x_{jpt}) \leq R_{kt}^{w} + R_{kt}^{z} \qquad (t=1,\ldots,T,\ k=1,\ldots,K) \quad (5)$$

In the (C1) criterion a penalty sum for projects in portfolio delays is minimized. A delay is a situation, when activity is finished later that latest finish time given from the CPM method (F_{jp}-LF_{jp}). A decision maker can also define latest time for activities completion. This criterion assumes that there is a grace period Y_p for project delays. During this period penalty is not charged.

The penalty cannot exceed a definite amount. A penalty function can be presented as follows (Figure 2).

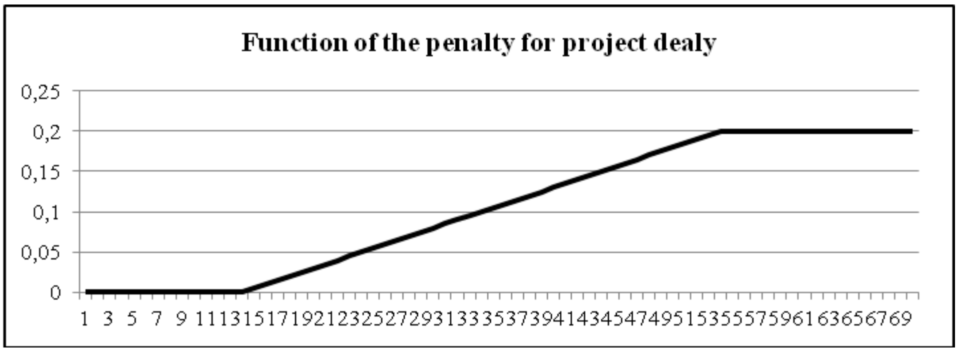

Figure 2. Penalty function

The purpose of the (C2) criterion is to minimize the cost of external resources usage. if the internal resources usage exceed the available amount, then a penalty for each unit is counted.

In the criterion (C3) NPV is maximized. This problem is used in the literature frequently. An example is presented in Icmeli and Erenguc (1996) paper. In this paper cash flows are generated in each unit of time of activity duration. This problem can be formulated as follows: $\sum_{i=1}^{J}[\sum_{t=1}^{d_j}[cf_{jt}\cdot e^{\alpha(d_j-t)}]\cdot e^{-\alpha F_j}]\rightarrow \max$.

In mathematical model presented in this paper we assumed that cash flows are generated by activities at the end of their duration and NPV is defined for project portfolio, so the criterion is formulated as follows: $\sum_{p=1}^{P}\sum_{j=1}^{J}cf_{jp}\cdot e^{-\alpha F_{jp}}\rightarrow \max$.

Binary variables are used in the considered model (1). The variable $x_{jpt}=1$ when an activity j last in time t, otherwise $x_{pjt}=0$. in the considered problem we have $J\times P\times T$ variables. Constraint (2) ensures appropriate activities duration. Finish times are used in criteria functions and in precedence relationships. Activities start times are necessary for precedence relationship constraints. That is why in equations (3) and (3') those times are determined. Constraint (3'') ensures that activity is not split. Precedence relationships are determined by constraint (4'). Constraint (5) is a resource one. Total resource usage should not exceed the sum on internal and external availability resources.

At the first phase of the methodology for project portfolio scheduling problem, mathematical model should be implemented to the problem described in this paper.

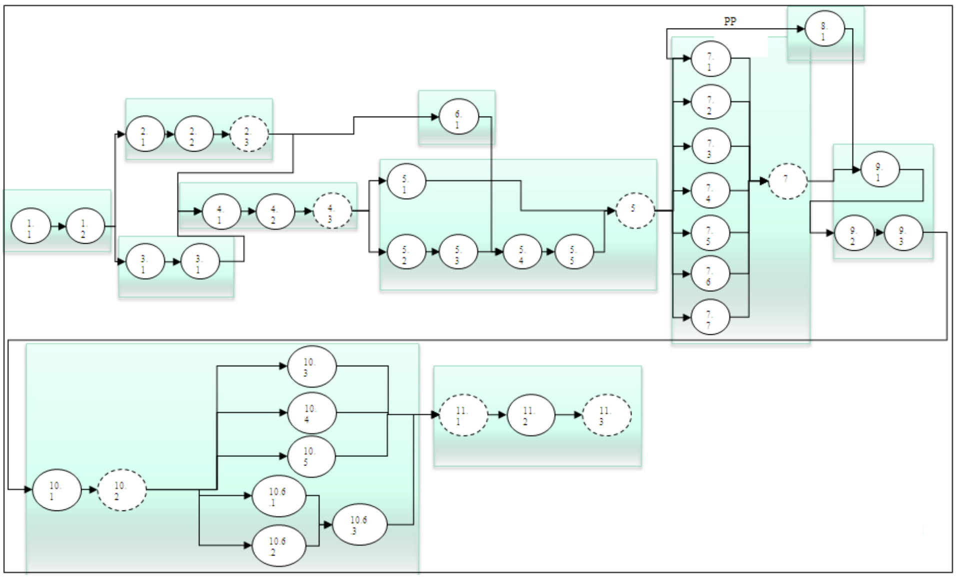

Figure 3. AON Network with activities group to aggregation

There are four projects in the project portfolio $P=4$. a single project has been described by 37 activities, $J=37$. Time planned for project portfolio realization is 252 (working days) $T=252$. Number of variables for this problem is $J\times P\times T=4\times 37\times 252=37296$. This number is huge, that is why activities aggregation will be used. We can reduce the number of variables three times by activities aggregation. In the given problem activities will be grouped into 11 phases (Figure 3, 4). After activities aggregation there is $J\times P\times T=4\times 11\times 252=11088$ activities. A schedule is a tool for project portfolio manager, for whom information about project phases is more important than information about single activity, reduced information caused by activity aggregation is not a disadvantage.

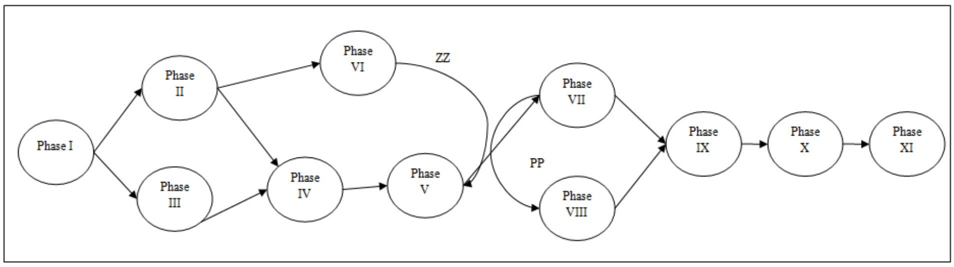

Figure 4. AON Network with aggregated activities

3.2 Finding non-dominated solutions

The second step of the methodology is to find the Pareto set. To find non-dominated solution an elastic evolutionary algorithm SPEA 2 (Zitzler, Laumanns, Thiele 2001) (Strength Pareto Evolutionary Approach) is used.

The main loop for SPEA2 algorithm can be presented as follows:
Step 1: Initialization.

Generate initial population P_0 and create the empty external set $\overline{P_0}$.

Step 2: Performance.

Fitness assignment for individuals in $P_{0\ and}$ $\overline{P_0}$.

Step 3: Selection and external set updating.

Copy All non-dominated solutions from $\overline{P_t}$ and P_t to $\overline{P_{t+1}}$.

If $\overline{P_{t+1}}$ exceeds the size of external set it should be reduced.

If the number of individuals in $\overline{P_{t+1}}$ is less the size of external set then it should be filled with dominated individuals from $\overline{P_t}$ and P_t.

Step 4: Termination.

If stop criterion is fulfilled then stop the algorithm. Individuals from external set $\overline{P_{t+1}}$ became a decision vectors.

Step 5: Mating selection.

A tournament selection with replacement on $\overline{P_{t+1}}$ to fill the mating pool.

Step 6: Variation.

Genetic operators are applied on individuals from the mating pool. the population P_{t+1} is a result of variation.

An elitist evolutionary algorithms use an external set, which is filled with the best solutions (in case of multiple criteria decision making problems there are non-dominated solution in external set). In each generation the external set is updated.

In step 1 an initial population is generated as a set of feasible solutions. At first, the projects sequences is randomized. Then activities are put to the empty schedule with appropriate sequence. an activity is schedule in the earliest possible time. Precedence relationships and resources availability is taken into account.

In step 2 values of objective functions are evaluated. There is a penalty, when an individual does not meet all constraints. Then, each individual from external set is assigned a strength value *S(i)* representing the number of individuals its dominate: $S(i) = |\{ j \mid j \in P_t + \overline{P_t} \wedge i \succ j \}|$. on the basis on this value a raw fitness of individual *i* is calculated $R(i) = \sum_{j \in P_t + \overline{P_t},\, j \succ i} S(j)$. to discriminate between individuals with the same raw fitness value a density information is incorporated. Density is defined by $D(i) = \frac{1}{\sigma_i^k + 2}$. An individual fitness is $F(i) = R(i) + D(i)$.

In step 3 an environmental selection is performed. if the number of individual fits exactly into the external set, then the environmental selection is completed. Otherwise, there can be two situations: the external set is too small or too large. When external set is too small, then dominated individuals in previous external set and population are copied to the new external set. When external set is too large, then it is reduced by individual removing. an individual with the smallest distance to another one is removed (Figure 5).

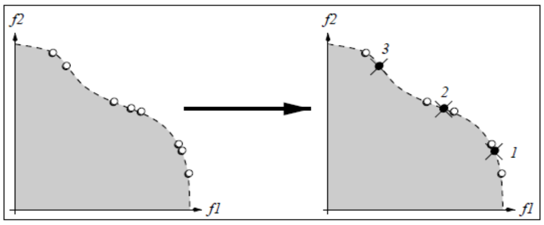

Figure 5. External set reduction in SPEA2 algorithm

Source: [11]

In the problem described in this paper binary variables are proposed. That is why an individual is a binary matrix with the projects and activities in rows and time periods in columns. the matrix size is $JP \times T$ and can be presented as follows:

$$i = \begin{bmatrix} x_{1,1,1} & x_{1,1,2} & \cdots & x_{1,1,T} \\ x_{2,1,1} & x_{2,1,2} & \cdots & x_{2,1,T} \\ \vdots & \vdots & \vdots & \vdots \\ x_{JP,1} & x_{JP,2} & \dots & x_{JPT} \end{bmatrix}$$

In step 6 genetic operators (crossover and mutation) are applied on individuals from the mating pool. a crossover is a process of exchanging a random row between two individuals. In mutation a random chosen activity is delayed.

3.3 Final solution choosing

A non-dominated set obtained in the second stage are used in the third stage of the methodology. in this stage the Light Beam Search (Jaszkiewicz, Roman 1999) with decision maker engagement is used to find the final solution.

A finite sample of non-dominated points is generated in each computation phase. This sample consists of middle point (obtained in previous iteration), and non-dominated solutions from its neighborhood.

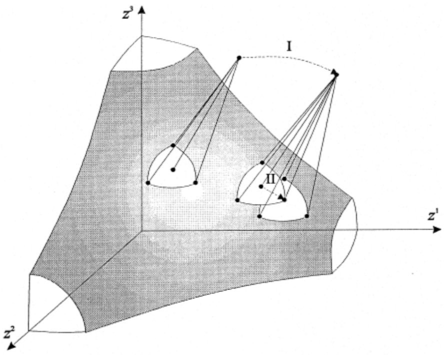

Figure 6. External set reduction

Source: Jaszkiewicz, Słowiński 1999.

A neighborhood of the middle point is a set of non-dominated points that are not worse than the middle point. In each phase of computation a sample of the solutions is presented to the decision maker. Decision maker can: make a point from the neighborhood to be a new middle point, define new aspiration and reservation points (starting aspiration and reservation points are used to define starting middle point), update preferential information, store point or decide that point found in the process is the final solution (Figure 6).

Conclusions

A new methodology for project portfolio scheduling problem has been presented in this paper. This methodology delivers information about project portfolio schedule, which is important for project portfolio managers.

The methodology consists of three steps: multiple-criteria mathematical model building, finding non-dominated solutions and choosing one solution from the Pareto set as the final solution.

In the first step a multiple-criteria mathematical model for project portfolio scheduling problem is built. Three criteria are considered in the model: the penalty for projects delays minimization, the penalty for resources overusage minimization and NPV maximization. Disadvantage of proposed mathematical model is a big amount of variables, which is $J \times T \times P$. In cases when we will have larger projects or larger planning horizon the number of variables will be huge. That is why activities aggregation was proposed. It reduces a number of variables in given model. It also delivers a general information about project portfolio schedule, which is more important for project portfolio managers.

In the second step a set of non-dominated solutions is identified by using an elitist evolutionary algorithm. As research shows using an external set with the best solutions in each generation increases an algorithm efficiency.

In the third step the LBS procedure is used to identify the final solution. The LBS procedure proposes is a way of learning-oriented interactive search for the best compromise solution for the decision maker. This procedure makes the comparison of non-dominated solution in the decision phase relatively easy. So this step will not burden the decision maker too much.

For the future work, the procedure presented in this paper will be applied to the problem described in the section two, and given results will be compared with the current situation in the company.

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