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SIMULATION AND OPTIMIZATION EXPERIMENTS ON SOME MODEL OF SD TYPE – ASPECT OF STABILITY AND CHAOS

Introduction

The problems of stability, bifurcation and chaos are nowadays in the center of the concern for many scientific disciplines, from the pure mathematics and control theory, to the practical experiences of management and economy. These problems are from their nature, interdisciplinary and they have many practical implications. Authors' experiences on the field of modelling, analysis and simulation the complex, non-linear, dynamic and multi-level systems, were developed from simple industrial models to the more sophisticated hybrid issues with using prof. Coyle's ideas so called "simulation during optimization" and "optimization during simulation" [Coyl96]. The aim of this paper is presentation of some new results authors' investigation in the area of simulation and optimization with use of the model of SD (System Dynamics [Coyl77, Coyl94, Coyl96, Coyl98, Forr61, Forr69, Forr71, Forr72, Forr75, Gonc08, Plat10, Rado01, Ster00, Ster02]) type. Especially the aspect of sensitivity analysis applied in stability and chaos searching, is the focus authors' interest. This topic is quite new in the field of using SD model to the analysis of the complex, nonlinear, dynamics and multi-level system. From many years the authors applied SD method in solving economics and management problems, using different hybrid issues and languages (see: publications [Kasp02, Kasp05, KaMa05, KaMa06, KaMS00, KaMS01, KaMS03, KaMS06, KaSl05]).

Only the latest experiments done on model DYNBALANCE (3-1-III) [KaMS06, Kasp09, KaSl06, KaSl05a, KaSl03], have allowed to underline the role of sensitivity analysis in the investigation of the stability and chaos in the modeled system. Only now with using language Vensim [Vens09] it is possible to planning the experiments of the searching stability and chaos of modeled systems. One of such system was created by Kasperska and named DYNBALANCE (3-1-III). It is the member of the "family" of models of the SD type (with hybrid additions), which was built by authors during last decade. On this model some experiments were undertaken and their results are presented in the paper.

Some theoretical issues

The changing surrounding us world causes that classic methods and tools are not sufficient for the analysis of dynamic, non-linear and complex systems. It should be stressed that in referring to such systems, one of the famous method named System Dynamics may be considered in 3 aspects of stability:

- stability of the initial conditions, that means for the vector of the initial values of levels (let mark it X_0);
- stability of the parameters of the model (let mark their vector by λ);
- stability of the structure (that means changing the vector of the function f from the system:

$$\begin{cases} \dot{X}(t) = f(X(t), t, \lambda, X_0) \\ X(t_0) = X_0 \end{cases}$$

In a theory of stability the main idea is of investigation of the sensitivity of trajectories of the state variables for the small perturbations. Therefore like we stress above, we can speak about:

- The perturbations of the initial conditions. Let mark distinguishing the initial state by:

$$\widetilde{X_0}(t_0) = \widetilde{X_0}$$

We obtain in Lapunow sense, the implication:

$$\left\|\widetilde{X_\delta}(t) - \widetilde{X_0}\right\| < \delta \Rightarrow \left\|\tilde{X}\left(t, t_0, \widetilde{X_\delta}, \lambda\right) - \tilde{X}(t, t_0, \widetilde{X_0}, \lambda\right\| < \varepsilon \qquad (1.2)$$

where $\|\circ\|$ is the euklidesion norm in R^m and $\delta, \varepsilon -$ small numbers in R. We can interpret this as follows: for the small differences of the initial value of the state vector we obtain the small differences of the trajectories in $R^m \times R$.

- Perturbation of the values of the parameters. In is this case the implication will be as follow:

$$\|\widetilde{\lambda_\delta} - \tilde{\lambda}\| < \delta => \|X(t, t_0, X_0, \widetilde{\lambda_\delta}) - X(t, t_0, X_0, \tilde{\lambda})\| < \varepsilon \qquad (1.3)$$

  Where $\tilde{\lambda}$ is the distinguishing vector of parameters. We can interpret this: for the small differences value of the parameters, we obtain a small (in sense of euklidesion norm) the differences of trajectories in $R^m \times R$.

- Perturbations of the structure (in sense of changing function f). It is showing the more interesting problem: what does it mean "small differences of structure"? How we measure this? The implication should be as follows:

$$\|f(X, t, \lambda_0, t_0) - \widetilde{f_\delta}(X, t, \lambda_0, t_0)\| < \delta => \\ \|X(t, t_0, \lambda_0, x_0) - X_{\tilde{f}}(t, t_0, \lambda_0, x_0)\| < \varepsilon \qquad (1.4)$$

In SD method functions f decide about the values of flows that feeds the levels. Like the consequences of changing f are the changing the values of rates (if the change is small we can talk about the structural stability). However the fact that in a system the functions f can be nonlinear and non-continuous, its implies that even small changing can cause the bifurcation, and the states of the state variables can be quite different. The problem is open. Still is the lack of the work (in SD field) on problem of stability, especially – structural stability. The modest effort, in such area of investigation, is further presented.

Presentation of the object of experiments and assumptions of the simulation type sensitivity analysis

The object of experiments was the model named DYNBALANCE (3-1-III) described in the literature of the field [Kasp09, KaMS06, KaSl03, KaSl05, KaSl06].

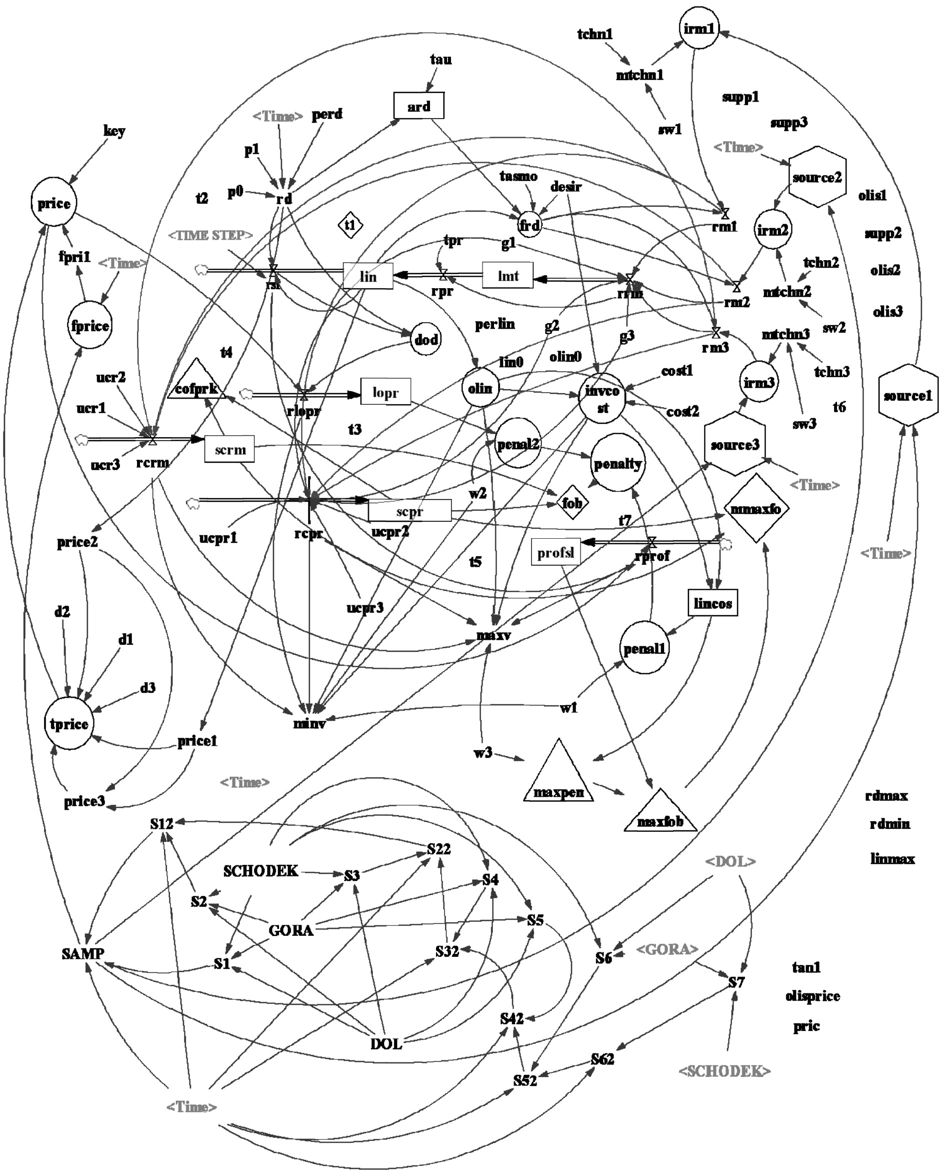

Fig. 1. The scheme of Vensim of model DYNBALANCE (3-1-III)

Source: Own idea.

The structure of this model, in Vensim [Vens09] convention, is presented above. Let summarized the main assumptions of the structure:

- the production is from three raw materials (*source*1, *source*2, *source*3),
- the transformation of raw materials into the product is modeled by a delay of order 3,

- the demand for the product has the sinusoidal form with two years faze and given amplitude,
- price of the product is the mixture of functions (2 options):
  - random sample function,
  - combination of 3 alternative elements; first depends on the state of the inventory of the product, second depends of the demand for the product and third is the minimum value of first and second,
- cost of the production is presented by function fob (measure the cost of raw materials and technology with people), with the penalty factor, modeled the cost of inventorying and the cost of losing profit (when the state of inventory was too small to satisfying the demand),
- the profit from the sale is summarizing, based on the actual price of the product.

Like the tool for experiments the Vensim language was chosen, because of his "sensitivity setup" and "optimization setup" [Vens09]. Now, let presented the assumptions of simulation experiments:

A) for the experiments type "sensitivity analysis", we first simulate the base conditions; the parameters: $tchn1, tchn2, tchn3$ (production productivities of three technologies from the raw materials ($source1, source2, source3$)), will be chosen (by Monte Carlo method) from the intervals [0,40]; the number of iteration will be 200,

B) for the experiments type "structural sensitivity", the parameter key (regulation-switch for the option for function $price$) will be chosen; first we assumed the interval for key like: [0,1] and then the interval: [0,0.1],

C) for the experiments type "optimization setup", fob like the objective function will be chosen, and like optimized parameters: $tchn1, tchn2, tchn3$, from the intervals [0,40] and then this intervals will become the objects of the "sensitivity analysis", and the sensitiveness for the scope of intervals for choosing value of the objective function will be searching,

D) for the experiments type "sensitiveness for the initial conditions", the initial value of lin (level of inventory) will be chosen, the sensitiveness of some variables will be investigated.

Time for the experiments.

Some chosen experiments on model DYNBALANCE (3-1-III) – aspect of stability and chaos

All the experiments were performed by applying language Vensim, version 5.

Experiment 1

In Figure 2 the results of simulation type “sensitivity analysis” are presented (see: the assumption A in chapter 2 of this paper). We can see how the model is sensitive for the values of parameters: $tchn1, tchn2, tchn3$ (using simultaneously). Of course for the investigation type stability (see: chapter 1) we should narrow the intervals of the values of the parameters. In Figure 3 such narrowing for parameter $tchn1$ (to the scope: [19,21]) is presented. We can assert that parameter $tchn1$ get stable results for variable fob (“cost of production”).

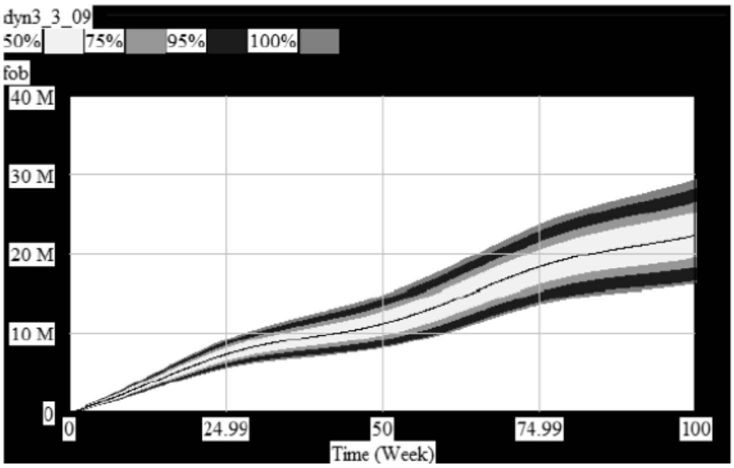

Fig. 2. “Confidence bounds” for variable fob

Source: Own results.

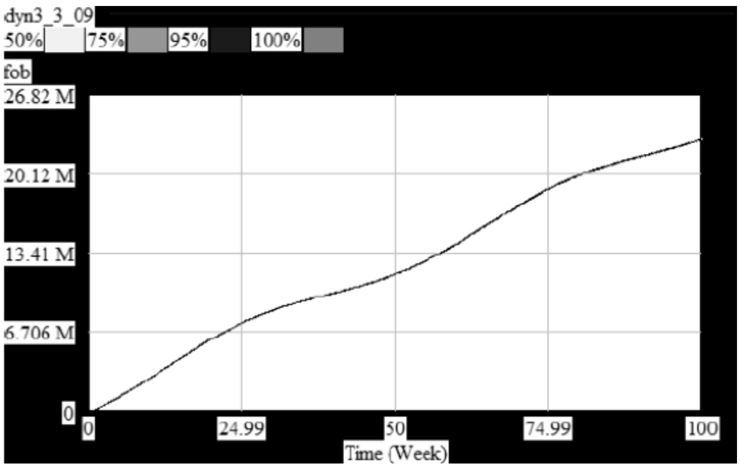

Fig. 3. The results of narrowing interval for $tchn1$

Source: Own results.

Experiment 2

In Figure 4 the results of simulation type "structural sensitivity" is presented (see: the assumption B in chapter 2 of this paper). For the experiment type "structural stability" we narrowed the interval for key, to the [0,0.1]. The results are presented in Figure 5. We can stress that the bifurcation behavior of variable $price$ occurs, like the results of switching the values of structural parameter key. So the system is structurally unstable.

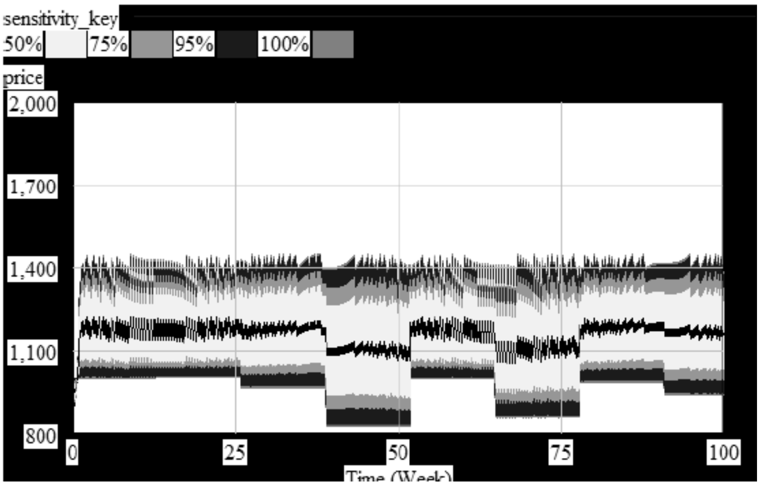

Fig. 4. "Confidence bounds" for variable $price$ with key [0.1]

Source: Own results.

Experiment 3

In Figure 6, 7, the results of "optimization experiment" type "sensitivity analysis" is presented (see: the assumption C in chapter 2 of this paper). First we have performed the optimization experiment type the minimization of cost (fob), getting the results shown in Figure 6, and then we established the scopes for intervals for optimized parameters like:

– $(0, tchn1opty)$,
– $(0, tchn2opty)$,
– $(0, tchn3opty)$.
– and performed the experiment type "sensitivity analysis", using Monte Carlo method by Vensim.

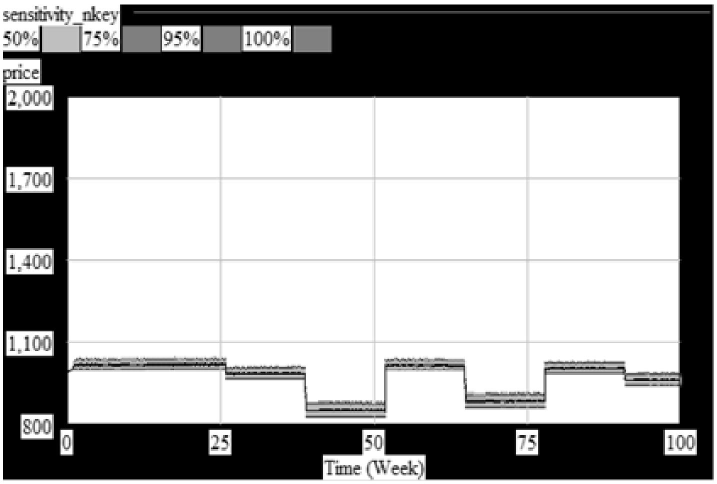

Fig. 5. Structural stability for variable $price$ using structural parameter key from interval [0,0.1]

Source: Own results.

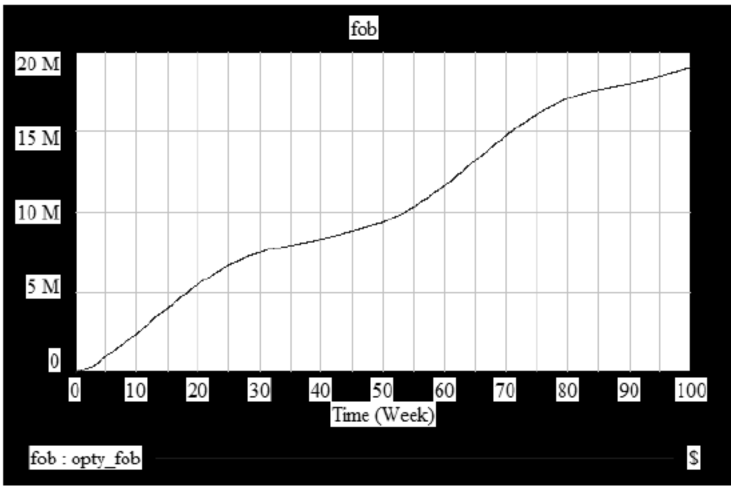

Fig. 6. Dynamics characteristic of variable fob for optimized parameters: $tchn1, tchn2$, $tchn3$

Source: Own results.

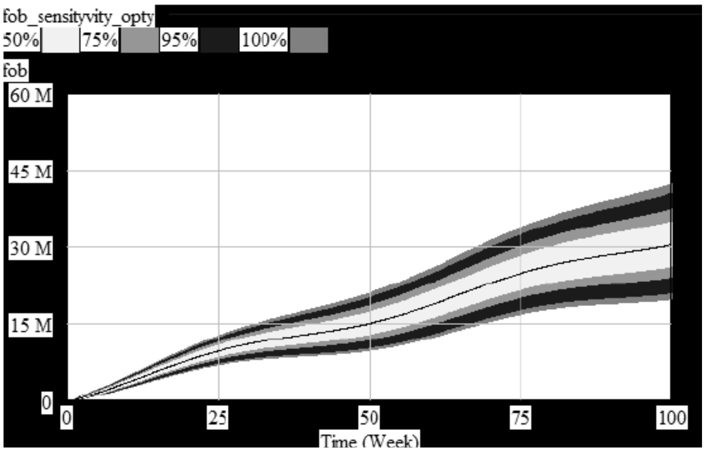

Fig. 7. "Confidence bounds" for variable fob using optimized values of parameters: $tchn1, tchn2, tchn3$

Source: Own results.

Experiment 4

In Figure 8 and 9 the results of simulation type "sensitiveness of initial conditions" is presented (see: the assumption D). We establish the scope of interval for $lin0$, and observe the results of using Monte Carlo method, for variables $price$ and lin.

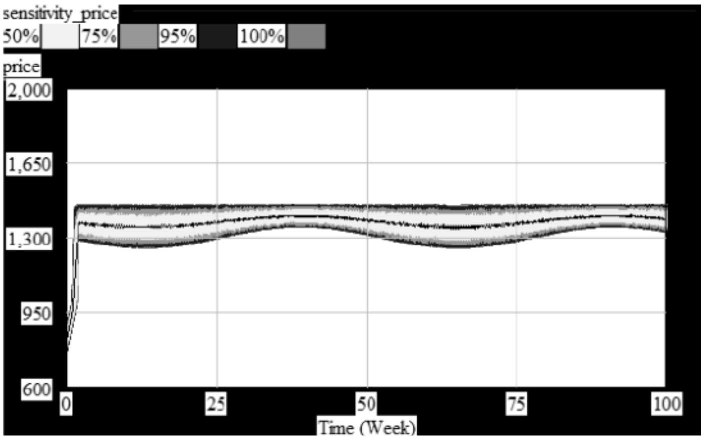

Fig. 8. "Confidence bounds" for variable $price$

Source: Own results.

Let finish the experimental series by the experiment number 5, which will be considered the chaotic behavior of some variable of the model DYNBALANCE (3-1-III).

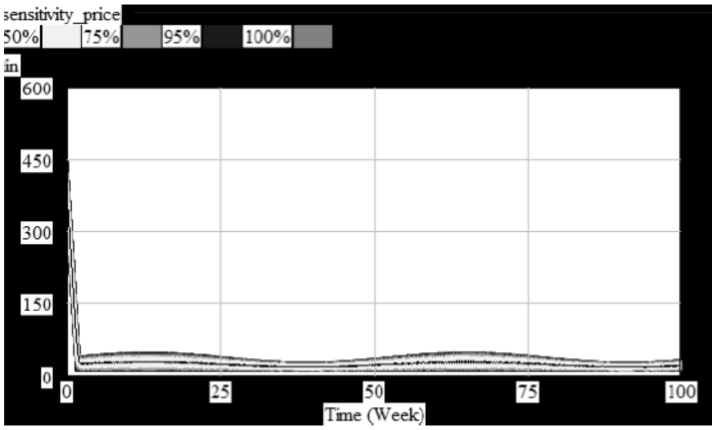

Fig. 9. "Confidence bounds" for variable lin

Source: Own results.

Experiment 5

Chaotic behavior occurs like the unstable changes in the characteristics of some variable like the answer on small changes in the initial condition. Let perform the simulation experiment, getting in particularly $key = 1$, $d2 = 0$, $d1 = 1$, and observe the characteristic of sale: rsl in the model. The results of experiment are shown in Figure 10, below.

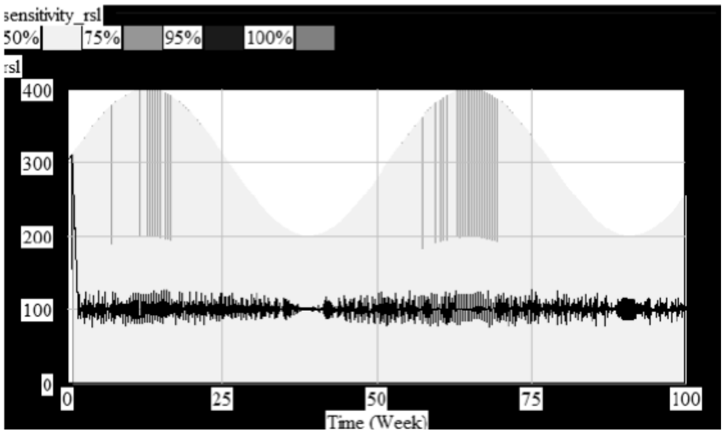

Fig. 10. The chaotic behavior of variable rsl, like the answer for changing the initial condition of lin

Source: Own results.

Final remarks and conclusions

Like the object of experimental investigation, the model DYNBALANCE (3-1-III) was chosen. The performed simulations have shown that the behavior of model is rather stable, under the considered conditions in the entrance ($source1$, $source2$, $source3$). But the structure of relations related to the sale causes some kind of instability in model. The experiments type sensitivity analysis can help in the investigation of such the behaviors and especially in the area of the optimization there are many possibilities of choosing the scopes of the parameters and types of the objective functions. The aim of the paper was presentation of some new results of authors' investigation in the area of simulation and optimization with the use of the model of System Dynamics type. Especially the aspect of sensitivity analysis applied in stability and chaos searching was the focus authors' interest. This topic was quite new in the field of SD and needs farther research efforts.

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EKSPERYMENTY SYMULACYJNE I OPTYMALIZACYJNE NA MODELU TYPU SD – PROBLEM STABILNOŚCI I CHAOSU

Streszczenie

Problemy stabilności i chaosu są obecnie w centrum zainteresowań wielu dyscyplin naukowych, od matematyki i teorii sterowania począwszy, po praktyczne zagadnienia zarządzania i ekonomii. Problemy te są ze swej natury interdyscyplinarne i mają wiele praktycznych zastosowań. Doświadczenia autorów w dziedzinie modelowania, analizy i symulacji złożonych, nieliniowych, dynamicznych i wielopoziomowych systemów rozwijały się stopniowo od prostych przemysłowych modeli poprzez bardziej złożone hybrydowe zastosowania według idei prof. Coyle'a zwanych w literaturze przedmiotu „symulacją zanurzoną w optymalizację" oraz „optymalizacją zanurzoną w symulację". Celem tego artykułu jest przedstawienie nowych wyników otrzymanych przez autorów w dziedzinie symulacji i optymalizacji na modelach typu SD (Dynamiki Systemowej [Coyl77, Coyl94, Coyl96, Coyl98, Forr61, Forr69, Forr71, Forr72, Forr75, Gonc08, Plat10, Rado01, Ster00, Ster02]). W szczególności zajęto się aspektem „analizy wrażliwości" w badaniu stabilności i chaosu. Jest to nowe zagadnienie w dziedzinie zastosowania modeli typu SD dla analizy złożonych, nieliniowych, dynamicznych i wielopoziomowych systemów. Przez wiele lat autorzy stosowali metodę SD przy rozwiązywaniu problemów ekonomii i zarządzania, używając różnych zastosowań i języków [patrz publikacje: Kasp02, Kasp05, KaMa05, KaMa06, KaMS00, KaMS01, KaMS03, KaMS06, KaSl05].

Ostatnie badania na modelu DYNBALANCE (3-1-III) [Kasp09, KaMS06, KaSl03, KaSl05a, KaSl06] pozwalają na podkreślenie istotnej roli „analizy wrażliwości" w badaniu stabilności i chaosu modelowanego systemu. W tym artykule zaprezentowano niektóre wyniki przeprowadzonej analizy.