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INTERACTIVE APPROACH AND ITS APPLICATIONS IN MANAGERIAL DECISION MAKING PROBLEMS

1. Introduction

Most of the real-world decision making problems involve multiple and conflicting objectives. During the last 40 years numerous techniques have been proposed in multiple criteria decision making. In practice, two main approaches are usually used for solving multiple criteria problems. The first one is based on a priori information on the decision maker's preferences. In this case the decision problem is solved in two steps. First, preference information is collected. Next, this information is used for constructing complete or partial order of the alternatives. Several criticisms have been expressed against this approach. First, the assessment of the sufficient a priori information about decision maker's preferences is inconvenient and time consuming. If the decision problem is repetitive then the decision maker's preferences can be inferred from the past decisions. Usually, however, direct questioning technique has to be employed. Thus, the decision maker is asked to make the hypothetical choices between alternatives that often have no practical reality. It is not easy to motivate the decision maker to consider and evaluate such choices. Moreover, as the decision maker is not employed in the second phase of the procedure, when the final solution is generated, so he/she may feel excluded from the important part of the analysis and put little confidence in a final result.

Interactive approach is opposite to the techniques based on an a priori basis. Instead of collecting all the preference information from the decision maker prior to calculating the final solution, this information is obtained by a stepwise procedure. The decision maker is asked to define which attributes (goal variables) influence his/her preferences and to provide preference information with respect to a given solution or a given set of solutions (local preference information). Two main advantages are usually mentioned for employing the interactive techniques. First, such methods need much less a priori information on the decision maker's preferences. Second, as the decision maker actively participates in all phases of the problem solving process, so he/she puts much reliance in the generated solution, and as a result, the final solution has a better chance of being implemented.

In this paper the main ideas of the interactive approach are presented. The paper is organized as follows. A multiple criteria decision making problem is defined in section 2. Section 3 presents a brief survey of the interactive techniques used for solving decision making problems under certainty. Next section is devoted to the stochastic dominance rules that can be used for comparing the uncertain projects. In section 5 the interactive procedures for the discrete stochastic multiple criteria decision making problems are proposed. Applications of these techniques in managerial decision making problems are discussed in section 6. The last section groups conclusions.

2. Multiple criteria decision making problem

The decision making problem, in its most general formulation, may take the form of the following statement (Kaliszewski, Michałowski, 1999):

"Given a set of decisions choose the best one according

to the decision making circumstances."

In practice, most of real-world decision making problems involve multiple criteria. In such a case the decision problem is defined by a set of alternative actions, a set of criteria, and a set of evaluations of the actions with respect to criteria. Such a description is quite general and comprises a wide range of the various decision situations. As a result, numerous techniques are proposed for solving the various types of multiple criteria decision making problems. In order to choose the method that might be beneficial in a particular situation, one should first determine the most important characteristics of the problem. These characteristics describe the data that are available, the information processing system, and the required output (Spronk, 1981).

Two alternate formulations of the problem can be distinguished considering the way in which the set of actions is specified. If this set is described explicitly (one by one), the decision problem is referred to as a discrete multiple criteria decision making problem. If, however, the set of the actions is defined implicitly (by means of constraints), the problem is refereed to as a continuous multiple criteria optimization problem.

This paper focuses on the discrete decision making problems. In this case the decision making situation can be described by an A-X-E model, where A is the set of actions, X is the set of attributes, and E is the performance table:

$$\mathbf{A} = \{a_1, \dots, a_i, \dots, a_m\}$$
$$\mathbf{X} = \{X_1, \dots, X_j, \dots, X_n\}$$
$$\mathbf{E} = \begin{array}{c|ccccc} & X_1 & \cdots & X_j & \cdots & X_n \\ \hline a_1 & e_{11} & \cdots & e_{1j} & \cdots & e_{1n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_i & e_{i1} & \cdots & e_{ij} & \cdots & e_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_m & e_{m1} & \cdots & e_{mj} & \cdots & e_{mn} \end{array}$$

We assume that larger values of attributes are preferred to smaller ones. Various types of decision problems can be analyzed considering the nature of the actions' evaluations. If these evaluations take form of real numbers, the problem is referred to as a deterministic decision making problem. In the case of evaluations handled as probability distributions, the problem is referred to as a stochastic decision making problem. Finally, a fuzzy decision making problem is considered if the evaluations are formulated as fuzzy numbers. Mixed problems can also be analyzed. In this case the evaluations of actions with respect to the various attributes differ in nature.

3. A brief review of the interactive techniques

The numerous interactive techniques have been proposed for the last 35 years. Procedures that are proposed differ in the way in which the communication process between the decision maker, an analyst and the model constructed by the analyst in consultation with the decision maker. There are, however, common features that characterize all these methods. In the interactive approach the decision maker has to express his/her preferences with respect to a single action or a small subset of actions, which are presented to him/her. Usually an initial solution is proposed to the decision maker. The latter expresses his/her preference information with respect to this solution to the analyst, who uses it for generating a new solution. Such a procedure continues until a satisfactory solution is obtained. Thus, the interactive approach corresponds to the Simon's theory of "satisficing". He noticed that managers are usually focused on finding "satisfactory" solution rather then "optimal" solution (Simon, 1957).

Two main issues have to be considered when an interactive procedure is designed: the way in which the communication process between the decision maker and the model is structured and the way in which the local preference information provided by the decision maker is used for generating new solutions. Obviously, the first issue is of crucial importance for the acceptance of the interactive technique by the decision maker. On one hand, sufficient information should be provided to the decision maker, on the other, the way in he/she is able to express his/her preferences should also satisfy him/her.

First interactive procedures have been proposed in 1970s. In STEM (Step Method) proposed by Benayoun et al. (1971) the concept of ideal solution is used. The elements of the ideal solution are the maximum values of the attributes, which are individually attainable within the set of actions. STEM is based on the calculation of a candidate action, which has a minimal distance to the ideal solution according to the mini-max rule. If the decision maker accepts the proposal, then the procedure ends, otherwise the decision maker is asked to define the amounts of relaxation for these attributes, whose values are already satisfactory. Then a new set of actions is generated taking into account the restrictions defined by the decision maker. The procedure continues until an action with satisfactory attribute evaluations is found.

A number of techniques based on the trade-off ratios are proposed. The classical method by Geoffrion et al. (1972) is one of them. In this method the decision maker has to determine the trade-offs between attributes at each iteration, given the attributes' values for the considered action. It is assumed that the decision maker's preferences can be described by a differentiable, concave and increasing utility function. As, however, this function is unknown, so the decision maker is asked to provide the information on trade-offs. Each iteration consists of two steps. First, the direction of the utility function fastest growth is calculated, next, the distance by which the solution should be moved in this direction is generated. The latter task is realized in the interaction with the decision maker. A number of techniques based on the same concept have been proposed. A method presented by Dyer (1972) is an example. He adopted trade-offs approach for the one-sided goal programming model. In other methods the decision maker has to specify an interval for each local trade-off ratio (e.g. Salo and Hämäläinen, 1992) or comparative trade-off ratios (e.g. Kaliszewski and Michałowski, 1997).

Another group of techniques consists of the methods based on pairwise comparisons. This approach is used, for example, by Zionts and Wallenius (1976). In this procedure a proposed action and neighboring actions are considered at each iteration. The information on the substitution between attributes for the proposed action and neighboring actions is presented. The decision maker is asked to decide whether he/she accepts moves from the proposed action to neighboring actions. The possible answers are: YES, NO, I DON'T KNOW. If at least one answer is YES, a new proposal is generated, otherwise, the considered action is assumed to be satisfactory for the decision maker.

The procedure proposed by Steuer (1986) is based on the vector comparisons. This technique is used for solving multiple criteria linear problems with a linear additive utility function as an objective function. As weighting coefficients defining this function are unknown, so the problem is not trivial. At each iteration the decision maker is presented a small subset of actions and is asked to choose the one that he/she prefers. This preference information is used for generating a new subset of the considered actions. The procedure continues until the satisfactory action is generated.

Methods based on manipulation of points of reference ones are also proposed. Such approach has been proposed, for example, by Wierzbicki (1980), Zeleny (1982), Korhonen and Laakso (1986), Michalowski and Szapiro (1992).

Another class of the interactive methods consists of techniques in which the decision maker has to define minimum or maximum values for one or more goal variables at each iteration. These restrictions are then used to reduce the feasible region. Such approach is used in an interactive multiple goal programming method proposed by Spronk (1981). In his procedure a proposal solution and potency matrix is presented to the decision maker. The solution is a vector of minimum values for the respective goal variables. The potency matrix consists of two vectors representing the ideal and pessimistic solution, respectively. If the proposal solution is not satisfactory for the decision maker, then he/she is asked to choose the goal variable, that should be improved first. The decision maker does not have to specify the amount by which the considered goal variable should be improved. If however he/she is able to provide the information about aspiration levels for goal variables, then this kind of information can be used by the procedure.

Methods that combine one or more of the above approaches are also proposed. Kaliszewski and Michalowski (1999) propose NIDMA procedure, that allows the decision maker to use different search principles depending on his/her perception of the achieved values of attributes and trade-offs. Some of the discussed approaches have been adopted for discrete multicriteria decision making problems. Procedures for such problems have been proposed, for example, by Roy (1976), Spronk (1981), Zionts (1981), Korhonen et al. (1984), Lotfi et al. (1992), Habenicht (1992), Sun and Steuer (1996), Trzaskalik (1998).

The procedures mentioned above are designed for the decision making problems under certainty. In practice most of the decision situations involve uncertainty. Different decision 'contexts', depending on what is known about the decision maker's preference structures and probability anticipations are considered in the literature (see Ben Abdelaziz et al., 1999). The numerous techniques have been proposed for solving the multiobjective stochastic linear programming problems. Some of them adopt interactive methodology for selecting the final solution. This approach is employed, for example, in the PROTRADE method of Goicoechea et al. (1982), in the Strage method of Teghem et al. (1986), in PROMISE/scenarios method of Urli and Nadeau (2004), and in a fuzzy satisficing method proposed by Sakawa et al. (2003).

4. The stochastic Dominance Rules

Two main approaches are usually used for the comparing uncertain prospects. The first is known as a mean-risk model and basis on two criteria: one measuring expected outcome and the second representing variability of outcomes. The mean-risk analysis allows to model preferences of a risk-averse decision-maker. Although the model of risk-averse preferences is widely exploited in decision theory, it is not suitable for all situations. Markowitz (1952) noticed the occurrence of risk seeking in choices between negative prospects. This paradox was also justified by the experiments conducted by Kahneman and Tversky (1979). Moreover, mean-risk approaches are not capable of modeling even the entire gamut of risk-averse preferences (Ogryczak and Ruszczyński, 1999).

The second approach uses the stochastic dominance rules. Two groups of the stochastic dominance relations can be considered. The first one includes FSD, SSD, and TSD, which means first, second, and third degree stochastic dominance respectively. These rules can be applied for modeling the risk averse preferences. The second group includes FSD and three types of inverse stochastic dominance: SISD, TISD1, TISD2 – second degree inverse stochastic dominance and third degree inverse stochastic dominance of the first and second type. These types of stochastic dominance rules can be applied for modeling risk seeking preferences.

Let us assume the following notation:
X, Y – random variables representing the outcomes of two uncertain prospects,
$F_X(\eta), F_Y(\eta)$ – right-continuous cumulative probability distributions:

$$F_X(\eta) = \Pr(X \le \eta) \qquad F_Y(\eta) = \Pr(Y \le \eta)$$

$\overline{F}_X(\eta), \overline{F}_Y(\eta)$ – left-continuous decumulative probability distributions:

$$\overline{F}_X(\eta) = \Pr(X \geq \eta) \qquad \overline{F}_Y(\eta) = \Pr(Y \geq \eta)$$

Stochastic dominance relations are defined in a following way:

Definition 1 (FSD – *First Degree Stochastic Dominance*):

$X \succ_{FSD} Y$ if and only if

$F_X(\eta) \neq F_Y(\eta)$ and $H_1(\eta) = F_X(\eta) - F_Y(\eta) \leq 0$ for all $\eta \in [a,b]$

Definition 2 (SSD – *Second Degree Stochastic Dominance*):

$X \succ_{SSD} Y(x)$ if and only if

$$F_X(\eta) \neq F_Y(\eta) \text{ and } H_2(\eta) = \int_a^\eta H_1(\xi)d\xi \leq 0 \text{ for all } \eta \in [a,b]$$

Definition 3 (TSD – *Third Degree Stochastic Dominance*):

$X \succ_{TSD} Y$ if and only if

$$F_X(\eta) \neq F_Y(\eta) \text{ and } H_3(\eta) = \int_a^\eta H_2(\xi)d\xi \leq 0 \text{ for all } \eta \in [a,b]$$

Definition 4 (SISD – *Second Degree Inverse Stochastic Dominance*):

$X \succ_{SISD} Y$ if and only if

$$\overline{F}_X(\eta) \neq \overline{F}_Y(\eta) \text{ and } \overline{H}_2(\eta) = \int_\eta^b \overline{H}_1(\xi)d\xi \geq 0 \text{ for all } \eta \in [a,b]$$

where: $\overline{H}_1(\eta) = \overline{F}_X(\eta) - \overline{F}_Y(\eta)$

Definition 5(TISD1 – *Third Degree Inverse Stochastic Dominance of the first type*):

$X \succ_{TISD1} Y$ if and only if

$$\overline{F}_X(\eta) \neq \overline{F}_Y(\eta) \text{ and } \overline{H}_3(\eta) = \int_\eta^b \overline{H}_2(\xi)d\xi \geq 0 \text{ for all } \eta \in [a,b]$$

Definition 6 (TISD1 – *Third Degree Inverse Stochastic Dominance of the second type*):

$X \succ_{TISD2} Y$ if and only if

$$\overline{F}_X(\eta) \neq \overline{F}_Y(\eta) \text{ and } \widetilde{H}_3(\eta) = \int_a^\eta \overline{H}_2(\xi)d\xi \geq 0 \text{ for all } \eta \in [a,b]$$

The FSD rule can be applied in the case of an increasing utility function, i.e. for function $u(x)$ for which $u'(x) > 0$ for all x. The SSD rule can be applied for concave increasing utility function: $u'(x) > 0$ and $u''(x) \leq 0$. Finally, the TSD rule is for decreasing absolute risk aversion (DARA) utility function, i.e. for function with $u'(x) > 0$, $u''(x) \leq 0$, $u'''(x) \geq 0$ and $u'''(x) \cdot u'(x) \geq [u''(x)]^2$. The second group includes FSD and three types of the inverse stochastic dominance: SISD, TISD1, TISD2 – second degree inverse stochastic dominance and third degree inverse stochastic dominance of the first and second type. These types of the stochastic dominance rules can be applied for modeling risk seeking preferences. The SISD rule is limited to utility function: $u' > 0$ and $u'' \geq 0$, while TISD1 and TISD2 rules can be used in the case of increasing absolute risk aversion (INARA) utility function, i.e. function with $u'(x) > 0$, $u''(x) \geq 0$ and $u'''(x) \leq 0$ or $u'''(x) \geq 0$ and $u'''(x) \cdot u'(x) \leq [u''(x)]^2$. Taking into account the results of Kahneman and Tversky (1979), Zaras and Martel (1994) suggest using FSD-SSD-TSD rules when the problem is defined in the domain of gains, and FSD-SISD-TISD1-TISD2 rules in the domain of loses.

5. The Interactive Techniques For Stochastic The Discrete Multiple Criteria Decision Making Problems

This section considers a stochastic discrete multiple criteria decision making problem. In this case the evaluations of actions with respect to the attributes are described by probability distributions. As a result the comparison of two actions leads to the comparison of two vectors of probability distributions.

Various concepts for solving such a problem have been proposed. Keeney and Raiffa (1976) suggest the multiattribute utility function approach. They show that if the additive independence condition is verified, the multiattribute comparison of two actions can be decomposed to one-attribute comparisons. Thus, estimating one-attribute utility functions and assessing the form of the aggregate function can solve the problem. In practice, however, both estimating one-attribute utility functions and assessment of the aggregate function are difficult.

Huang et al. (1978) suggest modeling global preferences by employing multiattribute stochastic dominance rule. According to this rule action a_i is at least as good as a_j if evaluations of a_i dominate corresponding evaluations of a_j in relation to each attribute. Unfortunately, this rule is very rarely verified. Zaras and Martel (1994) suggest weakening the unanimity condition and accepting a majority attribute condition. They propose MSD_r – multiattribute stochastic dominance for a reduced number of attributes. This approach is based on the observation that people tend to simplify the multiattribute problem by taking into account only the most important attributes. The procedure consists of two steps. First, stochastic dominance relations are verified for each pair of actions with respect to all attributes. Next, multiattribute aggregation procedure based on the ELECTRE I methodology is used for generating the final ranking of actions. Initially actions are classified considering all attributes. If the ranking is not clear enough then new order is constructed taking all but least important attributes into account. The procedure proceeds until the decision maker accepts the ranking and the majority condition is valid.

Such approach can be applied if the decision maker's preferences have been expressed explicitly enough, i.e. it is possible to specify weighting coefficient expressing the importance of each attribute. The assessment of such information is usually time consuming and inconvenient. As the decision maker usually finds such analysis as an unnecessary confusion, it is not easy to motivate him/her to take active part in it. On the other hand, applications of such approach are limited to problems that are relatively small. As a stochastic dominance test has to be verified for each pair of actions and for each attribute, so the solution of the problem with large number of actions may take quite a long time. In such cases, the interactive techniques might offer much help.

This section presents three interactive procedures for the discrete stochastic decision making problems. The proposed techniques differ in the way in which the conversation process is structured. The first one is an extension of the STEM method. In each step a candidate action, which has a minimal distance to the ideal solution, is generated. A min-max rule is used for measuring this distance. The decision maker examines evaluations of the candidate action with respect to attributes and selects the one that satisfies him/her. Then the limit of the concessions, which can be made on average evaluations with respect to this attribute, is defined. The procedure continues until a satisfactory solution is found.

The second procedure, called INSDECM combines two approaches used for comparing the uncertain actions: mean-risk analysis and stochastic dominance. It is assumed that the decision maker is able to express his/her requirements defining the restrictions based both on the average evaluations and on scalar risk measures.

The last technique employs the preference threshold concept. It is based on the assumption that decision maker's restrictions on average evaluations on a specified attribute can be weakened, if the analysis of the evaluations with respect to other attributes shows that it is worth. The procedure is constructed in such a way that avoids excluding the actions that are only slightly worse than a specified acceptance level, and simultaneously strictly preferred with respect to other criteria.

Let us assume following notation:

X_i^k – distributional evaluation of the action a_i with respect to attribute X_k,

μ_i^k – average evaluation of a_i with respect to X_k,

$\succ_{\mathrm{SD}}$ – stochastic dominance relation consistent with the decision maker's utility function.

Procedure 1 (Nowak, 2004b)

The idea of this technique comes from the STEM method (Benayoun et al., 1971). At each iteration, like in the STEM method, a candidate action a_i is generated. The stochastic dominance rules are used for doing this. The conversational phase is based on the analysis of average evaluations. The decision maker examines these evaluations of the candidate action a_i with respect to all attributes and selects the attribute with respect to which a_i is satisfactory for him/her. Then the decision maker defines the limit of the concessions, which can be made on average evaluations with respect to the considered attribute.

The operation of the procedure is as follows:

1. Identify stochastic dominance relations between actions' evaluations with respect to attributes, calculate average evaluations μ_i^k, $i = 1,..., m$, $k = 1,..., n$;
2. $l := 1$, $\mathbf{A}_1 := \mathbf{A}$, $\mathbf{K} := \{1, \ldots, n\}$;
3. Identify candidate action a_i:

$$a_i := \arg \min_{a_j \in \mathbf{A}_l} \max_{k \in \mathbf{K}} \{d_{jk}^l\}$$

   where d_{jk}^l is the number of actions $a_i \in \mathbf{A}_l$ such that the evaluation of a_i dominate the evaluation of a_j with respect to attribute X_k according to the stochastic dominance relation consistent with the decision maker's utility function:

$$d_{jk}^l = \operatorname{card} \mathbf{D}_{jk}^l$$
$$\mathbf{D}_{jk}^l = \{a_i : a_i \in \mathbf{A}_l ; \mathrm{X}_i^k \succ_{\mathrm{SD}} \mathrm{X}_j^k\}$$

   In the case of a tie choose any a_i minimising the value of $\max_{k \in \mathbf{K}} \{d_{jk}^l\}$.

4. Present the data to the decision maker:
   - the average evaluations of the candidate action a_i with respect to attributes – μ_i^k, $k = 1,..., n$,
   - the values of d_{ik}^l for $k = 1,..., n$,
   - the maximal average evaluations $\mu_k^{\max}$ for $k = 1,..., n$:

$$\mu_k^{\max} = \max_{i:a_i \in \mathbf{A}_l} \left\{\mu_i^k\right\}$$

5. Ask the decision maker whether he/she is satisfied with the candidate action. If the answer is YES – the final solution is action a_i – go to 9, else – go to 6.
6. Ask the decision maker whether the candidate action is satisfactory with respect to at least one attribute. If the answer is YES – go to 7, else – it is impossible to find an action with satisfactory attribute evaluations by the procedure – go to 9.
7. Ask the decision maker to select attribute with respect to which the candidate action is satisfactory for him/her, say attribute X_k and to define δ_k – the limit of concessions, which can be made on average evaluations of the attribute X_k.
8. Generate the set $\mathbf{A}_{l+1} := \left\{a_j : a_j \in \mathbf{A}_l, \mu_j^k \ge \delta_k\right\}$, assume $l := l + 1$, $\mathbf{K} := \mathbf{K} \setminus \{k\}$, if $\mathbf{K} = \varnothing$ then $\mathbf{K} := \{1, \ldots, n\}$, go to 3.
9. The end of the procedure.

K is the set of attributes that are considered when the candidate action a_i is selected. Once the decision maker accepts the evaluation of a_i with respect to X_k, attribute X_k is removed from **K**. If **K** is empty and satisfactory action has not been identified, then again all attributes are included in **K**. As the evaluations are probability distributions, so we are not able to generate candidate action in the same way as it is done in the STEM method. We apply the stochastic dominance rules instead: the distance from the ideal solution is measured by the number of actions with evaluations dominating the evaluation of the considered action according to stochastic dominance relation. Two types of data are presented to the decision maker. The number of evaluations dominating the evaluation of candidate action with respect to X_k provide the information on the position of the this action in a "ranking" of actions with respect to X_k constructed according to stochastic dominance rules. At the same time the average evaluation μ_i^k together with $\mu_k^{\max}$ provide the information about the distance between the best action with respect to X_k and the proposed action. Thus, the decision maker is able to evaluate the candidate action and decide whether he/she accepts the evaluation of the candidate action with respect to X_k. In order to define the limit of the concessions for attribute X_k the decision maker is asked to define minimal value of average evaluation for that attribute. Obviously, as the decision maker accepts the evaluation of a_i with respect to X_k, so δ_k should not be greater than μ_i^k.

Procedure 2 (Nowak, 2005a)

This procedure, called INSDECM, is based on both the stochastic dominance rules and mean-risk analysis approach. The first concept is used when actions are ordered with the respect to each attribute separately. The second one is employed in conversational phase of the procedure. As an interaction between the decision maker and the model includes presentation of the results and formulating additional restrictions by the decision maker, so the way in which the data are presented and the additional requirements are formulated have to be specified. Initially the decision maker is asked to define the set of data to be presented for each action in relation to each attribute. Thus, he/she has to choose the set of scalar measures for each attribute. It is assumed however, that the decision maker may change his/her mind while the procedure goes on, and specify other sets of measures in the successive iterations. For example, while initially the decision maker may be interested mainly in the expected outcomes, in subsequent phases of the procedure he/she may focus on risk measures' values. We propose to use following measures in the dialog phase of the procedure:

- average evaluation of a_i with respect to X_k – μ_i^k,
- standard deviation – σ_i^k,
- lower standard semideviation from a target value ψ – $\overline{\sigma}_i^k(\psi)$,
- upper standard semideviation from a target value ψ – $\widetilde{\sigma}_i^k(\psi)$,
- lower mean semideviation from a target value ψ – $\overline{\delta}_i^k(\psi)$,
- upper mean semideviation from a target value ψ – $\widetilde{\delta}_i^k(\psi)$,
- probability of getting the outcome not exceeding a target value ψ – $\Pr(X_i^k \le \psi) = \mathrm{F}_i^k(\psi)$,
- probability of getting the outcome not less then a target value ψ – $\Pr(X_i^k \ge \psi) = \overline{\mathrm{F}}_i^k(\psi)$.

The first measure is usually used as a criterion for expected outcome and the rest ones are used as risk measures in the various mean-risk models.

Once the data have been presented, the decision maker is asked to decide whether he/she accepts one of the considered actions as the final solution of the procedure. If the decision maker is not able to make a final choice, he/she is asked to define additional restrictions. These restrictions are formulated by the minimal or maximal values of scalar measures that are acceptable to the decision maker. Thus, taking into account attribute X_k the decision maker may specify additional requirements e.g. in the following way:

– mean not less then a specified target value ξ: $\mu_i^k \geq \xi$,
– standard deviation not greater then a specified value ξ: $\sigma_i^k \leq \xi$,
– lower standard semideviation from a target value ψ not greater then a specified value ξ: $\bar{\sigma}_i^k(\psi) \leq \xi$,
– upper standard semideviation from a target value ψ not less then a specified value ξ: $\tilde{\sigma}_i^k(\psi) \geq \xi$,
– lower mean semideviation from a target value ψ not greater then a specified value ξ: $\bar{\delta}_i^k(\psi) \leq \xi$,
– upper mean semideviation from a target value ψ not less then a specified value ξ: $\tilde{\delta}_i^k(\psi) \geq \xi$,
– probability of getting outcome not exceeding a specified target value ψ not greater then a specified value α: $\Pr(X_i^k \leq \psi) \leq \alpha$,
– probability of getting outcome not less then a specified target value ψ not less then a specified value α: $\Pr(X_i^k \geq \psi) \geq \alpha$.

An important question arises: are such restrictions always consistent with the stochastic dominance rules? Unfortunately the answer is negative. In fact only the first restriction is always consistent with the stochastic dominance rules. Let us assume that a restriction for attribute X_k has been defined. The restriction is inconsistent with the stochastic dominance rule for a pair (a_i, a_j) if following conditions are simultaneously fulfilled:

– the evaluation of ai with respect to Xk does not satisfy the restriction,
– the evaluation of aj with respect to Xk satisfies the restriction,
– $X_i^k \succ_{\mathrm{SD}} X_j^k$.

In INSDECM each time a new restriction is formulated by the decision maker the consistency with stochastic dominance rules is verified. For each pair (a_i, a_j) such that a_i does not satisfy the condition and a_j satisfies it, we check whether the evaluation of a_i dominates the evaluation of a_j according to stochastic dominance rules. In such a case the decision maker is asked to redefine the restriction. The various forms of redefinition are suggested to the decision maker. The proposals are constructed in such a way, that either both actions for which the inconsistency has been found are accepted, or both are rejected (for details see Nowak, 2005a). If the decision maker does not accept any of proposals he is asked to specify requirements in a quite different way. Once the consistency is obtained, the set of actions satisfying decision maker's requirements is generated. The best and the worst actions with the respect to each attribute according to the stochastic dominance rules and values of corresponding scalar measures are presented. The decision maker may either accept the new set of considered actions or return back to the previous step and try to weaken the restriction or define it in another way.

The operation of the procedure is given below.

1. Generate the set of efficient actions $\mathbf{A}'$.
2. Identify the stochastic dominance relations between efficient actions with respect to attributes.
3. Ask the decision maker to specify for each attribute scalar measures that he/she finds interesting.
4. Let $l = 1$, $\mathbf{B}_l = \mathbf{A}'$.
5. Order actions $a_i \in \mathbf{B}_l$ according to stochastic dominance rules with respect to attribute X_k, $k = 1,..., n$.
6. If l is equal to 1, than go to 9, otherwise go to 8.
7. Present the best and the worst actions with respect to X_k, for $k = 1,..., n$ to the decision maker.
8. Ask the decision maker whether he/she accepts the move from $\mathbf{B}_{l-1}$ to $\mathbf{B}_l$. If the answer is YES – go to 9, else assume $l = l - 1$ and go to 5.
9. Present $a_i \in \mathbf{B}_l$ to the decision maker. If the decision maker is able to choose final solution then the procedure ends, else go to 10.
10. Present the best and the worst actions with respect to X_k, for $k = 1,..., n$ to the decision maker.
11. Present the best and the worst actions with respect to X_k, for $k = 1,..., n$ and values of corresponding scalar measures to the decision maker.
12. Ask the decision maker to define a new restriction.
13. Verify the consistency of the condition defined by the decision maker with stochastic dominance rules. If the inconsistency has been found go to 14, else go to 15.
14. Present the decision maker the ways in which the restriction can be redefined and ask him/her to choose one of these suggestions. If he/she does not accept any proposal – go to 12, else – replace the restriction by the accepted proposal and go to 15.

15. Generate $\mathbf{B}_{l+1}$ – the set of actions $a_i \in \mathbf{B}_l$ satisfying the considered restriction. If $\mathbf{B}_{l+1} = \varnothing$ than notify the decision maker and go to 12, else go to 5.

Scalar measures other then listed above can also be employed by the procedure. It is also possible to define the decision maker's restriction in other ways than it is proposed in this work. It should be kept in mind however, that the proposed technique is based on the stochastic dominance rules, and as a result the consistency of the decision maker's restrictions with these rules should be provided.

Procedure 3 (Nowak, 2005b)

The main motivation for proposing this procedure is the aspiration for modeling decision maker’s preferences more precisely than it is possible, when the reservation level is formulated as a crisp value. It is assumed that the decision maker’s restrictions on average evaluations on a specified attribute can be weakened, if the analysis of the evaluations with respect to other attributes shows that it is worth. The procedure is constructed in such a way that avoids excluding the actions that are only slightly worse than a specified acceptance level, and simultaneously strictly preferred with respect to other criteria. This is realized by employing the preference threshold concept.

The idea of the thresholds has been developed and promoted by Roy (1985). He formulates fundamental partial comparability axiom, according to which preferences for every two actions a_i and a_j can be modelled by exactly one of following relations: indifference, weak preference, strict preference, and incomparability. Roy introduces also the outranking relation. It is assumed that action a_i outranks action a_j if there are enough reasons to admit that in the eyes of decision maker a_i is at least as good as a_j. The indifference threshold q is used to distinguish between situations of indifference and weak preference, while preference threshold p is employed to differentiate weak and strict preference. Veto threshold v is also defined to indicate situations when the difference between two alternatives with respect to one specified attribute negates any possible outranking relation indicated by other criteria.

Preference threshold for decision problems under risk is defined as follows (Nowak, 2004a):

$$p_k = \alpha_k^P \mu_{ik} + \beta_k^P$$

where α_k^P, β_k^P are constants.

Comparing a_i and a_j with respect to attribute X_k we consider following situations:

1. a_i is strictly preferred to a_j: $a_i \ \mathrm{P}_k \ a_j \Leftrightarrow X_i^k \succ_{\mathrm{SD}} X_j^k$ and $\mu_i^k \geq \mu_j^k + p_k\left(\mu_i^k\right)$,
2. a_j is strictly preferred to a_i: $a_j \ \mathrm{P}_k \ a_i \Leftrightarrow X_j^k \succ_{\mathrm{SD}} X_i^k$ and $\mu_j^k \geq \mu_i^k + p_k\left(\mu_j^k\right)$,
3. a_i is weakly preferred to a_j:
   $a_i \ \mathrm{Q}_k \ a_j \Leftrightarrow X_i^k \succ_{\mathrm{SD}} X_j^k$ and $\mu_j^k \leq \mu_i^k < \mu_j^k + p_k\left(\mu_i^k\right)$,
4. a_j is weakly preferred to a_i:
   $a_j \ \mathrm{Q}_k \ a_i \Leftrightarrow X_j^k \succ_{\mathrm{SD}} X_i^k$ and $\mu_i^k \leq \mu_j^k < \mu_i^k + p_k\left(\mu_j^k\right)$,
5. non-preference: $a_i \ \mathrm{N}_k \ a_j$ – otherwise.

The last situation groups two cases: indifference when probability distributions are exactly identical and the lack of information. Strict or weak preference is assumed if and only if the stochastic dominance test is verified. The value of the preference threshold is based on the average performances. We assume a strict preference if stochastic dominance rule is verified and the difference between average performances is sufficiently large. If the stochastic dominance test is verified but the difference between the average performances is small then weak preference is assumed.

The idea of this technique comes from the interactive multiple goal programming. The set of actions is progressively reduced while decision maker defines additional requirements. At the beginning the decision maker is asked to define preference threshold for each attribute. Next, at each iteration, he/she is confronted with the set of considered actions. The average evaluations with respect to attributes are presented for each action. If the decision maker is able to make a final choice then the procedure ends, otherwise he/she is asked to specify a reservation level for one specified attribute X_k – minimal acceptable value for average evaluation μ_k^*. New set of actions is constructed according to the following rules:

(a) include actions a_i such that $\mu_i^k \geq \mu_k^*$;

(b) include actions a_i such that $\mu_i^k < \mu_k^*$, but for at least one action a_j satisfying rule (a) following condition is fulfilled: $a_j \ \mathrm{N}_k \ a_i$;

(c) include actions a_i such that $\mu_i^k < \mu_k^*$, but for at least one action a_j satisfying rule (a) $a_j \ \mathrm{Q}_k \ a_i$ and for at least one attribute $X_{k'} \neq X_k$: $a_i \ \mathrm{P}_{k'} \ a_j$.

Thus, the new set of actions includes not only the actions satisfying restriction defined by the decision maker, but also some actions that are not strictly dominated by all the actions satisfying this restriction. The procedure continues until the satisfactory action is generated.

Let us introduce some concepts used by the procedure. Let $\mathbf{A}_l$ is the set of actions considered in iteration l, m_l is the number of actions considered in iteration l, and n is the number of attributes. The matrix of the average evaluations $\mathbf{M}_l$ is the matrix consisted of m_l rows and n columns. Each row of this matrix consists of the average evaluations of a specified action with respect to all attributes:

$$\mathbf{M}_l = [\mu_i^k] \quad \text{for } i \text{ such that } a_i \in \mathbf{A}_l \text{ and } k = 1,..., n$$

The potency matrix $\mathbf{S}_l$ is a matrix consisted of 2 rows and n columns. The first row contains minimal and the second one maximal average evaluations for each attribute:

$$\mathbf{S}_l = \begin{bmatrix} \mu_1^{\min} & \mu_2^{\min} & \cdots & \mu_n^{\min} \\ \mu_1^{\max} & \mu_2^{\max} & \cdots & \mu_n^{\max} \end{bmatrix}$$

where: $\mu_k^{\min} = \min_i \{\mu_i^k\}$, $\mu_k^{\max} = \max_i \{\mu_i^k\}$ for i such that $a_i \in \mathbf{A}_l$, $k = 1,..., n$

The operation of the procedure is as follows:

<u>Initial phase:</u>

1. Ask the decision maker to define the preference thresholds for attributes;
2. Calculate average evaluations of actions with respect to attributes μ_i^k;
3. l:= 1, $\mathbf{A}_1$:= $\mathbf{A}$.

<u>Iteration l:</u>

1. Construct the matrix of average evaluations $\mathbf{M}_l$ and present it to the decision maker. If the decision maker is able to choose final solution then stop the procedure, otherwise go to step 2.
2. Present the potency matrix $\mathbf{S}_l$ to the decision maker.
3. Ask the decision maker to choose the attribute X_k for which he/she wants to define reservation level $\overset{*}{\mu_k}$ $(\mu_k^{\min} < \overset{*}{\mu_k} < \mu_k^{\max})$.
4. Generate $\mathbf{A}_{l+1}$:

$$\mathbf{B}_l^1 = \left\{ a_i \in \mathbf{A}_l : \mu_i^k \geq \overset{*}{\mu_k} \right\}$$

$$\mathbf{B}_l^2 = \left\{ a_i \in \mathbf{A}_l \setminus \mathbf{B}_l^1 : \mu_i^k < \overset{*}{\mu_k}; \exists a_j \in \mathbf{B}_l^1 : a_j \; \mathrm{N}_k \; a_i \right\}$$

$$\mathbf{B}_l^3 = \left\{ a_i \in \left(\mathbf{A}_l \setminus \mathbf{B}_l^1\right) \setminus \mathbf{B}_l^2 : \mu_i^k < \overset{*}{\mu_k}; \exists a_j \in \mathbf{B}_l^1, \exists k' \neq k : a_j \; \mathrm{Q}_k \; a_i \text{ and } a_i \; \mathrm{P}_{k'} \; a_j \right\}$$

$$\mathbf{A}_{l+1} = \mathbf{B}_l^1 \cup \mathbf{B}_l^2 \cup \mathbf{B}_l^3$$

5. Construct the potency matrix $\mathbf{S}_{l+1}$ and present it to the decision maker. If the decision maker accepts the shift from $\mathbf{S}_l$ to $\mathbf{S}_{l+1}$ then assume $l = l+1$ and go to step 1, otherwise return to step 3.

The decision maker expresses his/her preferences specifying reservation levels. It should be noticed that the decision maker has the possibility to resign or redefine the restriction, if he/she finds that it leads to unacceptable worsening of evaluations with respect to other attributes. Such approach is also used in interactive multiple goal programming (Spronk, 1981). In the procedure proposed here, however, new ideas are also employed. First, uncertainty is taken into account by employing the stochastic dominance concept. Next, restrictions defined by the decision maker are not taken as an ultimatum.

Each restriction defined by the decision maker is analyzed and can be weakened if it is justifiable. There are two main reasons for employing such an approach. First, it is usually not easy for the decision maker to formulate his/her requirements precisely when the evaluations take form of probability distributions. On the one hand, an average evaluation is the measure that can be interpreted relatively easy, on the other, however, the analysis should not be reduced to the comparison of the average evaluations. By employing the stochastic dominance rules we can take into account uncertainty. If the average evaluation of a considered action is less then the minimal value defined by the decision maker, and, in the same time, there is at least one other action satisfying this condition, which is neither strictly nor weakly preferred to the considered action, then we accept the considered action and take it into account in the succeeding steps of the procedure. In practice rule (b) means that we accept the actions, that do not satisfy the restriction formulated by the decision maker, but are not stochastically dominated by all the actions satisfying this restriction.

The second reason for weakening the decision maker's restriction is the aspiration for taking into account the actions, that are only weakly preferred by at least some of actions satisfying this restriction, and on the other hand, are strictly preferred to these actions with respect to at least one other attribute. This is realized by applying rule (c).

The procedures presented above can be applied for solving various decision problems. The question that arises is: which one should be applied in a particular situation? The choice depends on the problem peculiarity and on the decision maker’s perception of the proposed technique. The first technique is probably less demanding for the decision maker. At each iteration a proposed action is presented to him/her. The only information that he/she has to provide is the information about the attribute according to which the proposal is acceptable for him/her and the limit of concessions. The way in which uncertainty factor is taken into account may be considered to be a disadvantage of this technique. To be true, the decision maker is not able to analyse uncertainty directly. On the other hand, however, the information on the number of actions with evaluations that dominate the estimation of the proposed action according to the stochastic dominance rules is presented to the decision maker. Thus, he/she is able to analyze the position of the considered action in a ranking constructed with the respect to a specified attribute.

If the decision maker is able to specify his/her demands specifying the constraints based on the average evaluations and/or scalar risk measures, INSDECM procedure may be very useful. In this case, however, the decision maker has to accept that he/she might be asked to revise his/her demands in order to achieve the consistency with stochastic dominance relations. Moreover, the decision maker is usually able to examine only limited amount of information while problem solving procedure goes on. Thus, the approach proposed in INSDECM is usually useful when the number of attributes is not too large.

The third procedure is probably the most demanding, as the decision maker has to define the preference thresholds for attributes. Thus, some amount of a priori preference information has to be collected to employ this technique. There are significant advantages of employing this method, however. The restriction formulated by the decision maker is not considered to be an ultimatum. An additional analysis is conducted to state whether an action that does not satisfy the decision maker's restriction should still be taken into account in following phases of the procedure.

6. Applications

Techniques that are proposed in this paper can be successfully applied in problems with up to the moderate number of actions (not more than hundreds). In this section three applications are presented. First, project selection problem is analyzed. Next, an application in inventory planning is discussed. Finally, an application in production process control problem is considered.

Project Selection Problem

The various elements are usually taken into account when an investment project is analyzed. The technical factors, environmental effects, social issues and financial profitability are of the significant importance. Thus, project selection is a typical example of a multiple criteria decision making problem. Financial evaluation of the project can be done by a net present value – NPV, payback period, profitability index – PI, internal rate of return – IRR. At the same time the technical, technology and marketing criteria often are also taken into account when investment project is undertaken. In practice the decision maker always is faced with the uncertainty when he/she tries to evaluate investment project. Criteria such as NPV or IRR are based on the predicted values of the interest rates, prices and demand for products. In technical and marketing evaluation of the projects qualitative factors are very important. As various experts can evaluate each project in different way the decision maker often obtains different recommendations not only by various criteria but also by the same criteria depending on the experts' notions.

Let us assume that only one project can be selected. Thus, the set of actions A consists of all the considered projects. Each project is evaluated with the respect to attributes such as NPV, ISS, PI, level of technical novelty, chances of success, e.t.c. There are two sources of data that can be used for evaluating the actions with respect to attributes: simulation and experts' valuations. A series of simulation experiments is conducted for each project. Thus, a sequence of observations for each alternative with respect to each attribute is obtained. These observations are used for constructing distributional evaluations. At the same time several experts are asked to evaluate projects with the respect to qualitative attributes. As a result series of evaluations are obtained for each project with the respect to each qualitative attribute. Again, these data are exploited for generating the distributional evaluations.

The procedure for solving such a problem consists of the following steps:
1. Simulation analysis of the considered projects with respect to the financial measures;
2. The collection of experts' opinions with the respect to the qualitative criteria used in project evaluation;
3. Generation of distributional evaluations of projects with respect to attributes;
4. Employing interactive procedure for project selection.

Inventory planning

Traditionally, inventory planning problems are considered as the single criteria optimization problems. It is usually assumed that the decision maker's goal is to minimize total costs including storage cost, transaction cost and cost of stock-outs. In practice, however, it is not easy to estimate all these quantities.

Instead inventory planning problem can be considered as a multiple criteria problem.

Let us consider a fixed order quantity inventory management described by a $\langle Q, r\rangle$ model. Under this system, a fixed, predetermined quantity Q of an inventoried item is ordered when the stock on hand reaches a level called the reorder point r. The problem that arises id: how to determine values of Q and r. In this section we show the way in which interactive procedures can be employed for solving this problem.

In this problem the set of actions consists of all possible couples $\langle Q, r\rangle$. The following attributes are taken into account: storage and carrying costs, stock-out demand, the number of orders. It is assumed that probability distributions for the demand and lead time are known. The procedure for solving such a problem is as follows:

1. Simulation analysis of the considered couples $\langle Q, r\rangle$;
2. Generation of the distributional evaluations of actions with respect to the attributes;
3. Determining values of Q and r in an interactive procedure.

Production Process Control

Let us consider a problem of production process control in a job-shop environment. We assume that Kanban method is used in production control. The work flow is controlled by Kanban cards. An operation can be performed if the appropriate card is free. Decision rules are used to determine the sequence in which operations are performed on the considered station. Production may proceed differently according to a lot size, number of Kanban cards used, and the decision rule for choosing the waiting job to process. Smaller lot-sizes will usually reduce work-in-progress, but can also enlarge the number of machine set-ups. Larger number of cards improves machine utilisation, but also may increase average stock size. The choice of the best triplet involving the Kanban lot size, the decision rule and the number of Kanban cards constitute a multiple criteria decision problem.

In a problem considered here the set of actions includes all triplets – the lot size, the number of kanban cards, and the decision rule. The set of attributes includes:

- makespan,
- average stock (work-in-progress) level,
- the number of set-ups.

Performances of each action with respect to the attributes are evaluated by distribution functions. The knowledge base used for construction of these functions may be obtained by using the simulation model of the process. A sequence of the simulations is carried out for each triplet. For each action with respect to each attribute we have a sequence of observations that can be used for construction of distribution functions.

Procedure for solving the decision problem is as follows:

1. Simulation analysis of each triplet;
2. Generation of distributional evaluations of actions with respect to attributes;
3. Employing interactive procedure for choosing the final solution.

Conclusions

Interactive approach is one of the leading methodologies in the multiple criteria decision making. Several motivations have been mentioned for implementing this approach. It is usually pointed out that limited amount of a priori preference information is required from the decision maker as compared to other techniques. The interactive procedure may be considered as a learning process. Observing the results of succeeding iterations of the procedure the decision maker extends his/her knowledge of the decision problem. On the other hand, as the decision maker actively participated in all phases of problem solving procedure, the decision maker puts much reliance on the final solution that is obtained. As a result, the solution of the procedure has a better chance of being implemented.

In the paper three interactive procedures for discrete stochastic multiple criteria decision problems are suggested. The proposed methodology combines two concepts: interactive approach and stochastic dominance.

Multiple criteria analysis based on stochastic dominance has been successfully applied in the decision analysis during last thirty years. Initially the investments and savings, portfolio diversification, option evaluation and portfolio insurance were the main areas of application. Since 1990 various new areas of employment of the stochastic dominance concept has been proposed: production process control, investment projects' evaluation, measuring the quality of life. The methodology proposed in this paper can be employed in all these fields.

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