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*Piotr Saluga*

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**VALUATION OF A PROJECT  
WITH AN OPTION-TO-TEMPORARILY-  
-SHUT-DOWN – THE MULTIPLICATIVE  
VS. ADDITIVE BINOMIAL MODEL**

## Introduction

The first application of the stochastic process to describe the fluctuations in stock prices was proposed by Louis Bachelier in his doctoral thesis in mathematics *Théorie de la spéculation*, defended at the University of Paris in 1900. Bachelier defined the probability of exercising the option and calculated its value assuming that the stock price movements are independent random variables with normal distribution (Arithmetic Brownian Motion, ABM). The resulting formula, however, was based on assumptions that the interest rate was zero and the stock price might be negative.

The stochastic process has taken a permanent part in physics through the work of Einstein [1905] and Smoluchowski [1906], and in mathematics through the work of Wiener [1923] and Lévy [1939]. Building on the ideas of Bachelier and Osborne [1959; 1962] and – independently – Samuelson [1965] modified the stochastic process by introducing the idea of Geometric Brownian Motion (GBM). The purpose of this modification was to make the process of price movements realistic. Under GBM prices cannot be negative therefore the Arithmetic Brownian Motion is followed then not by the price but its natural logarithm.

The idea that stock prices follow the Geometric Brownian Motion became the basis for valuation of financial options. In 1973, Black and Scholes [1973] suggested a „bond – stock” market model in the form of two equations, the first of which was a deterministic equation with the value of bonds or bank deposits (with fixed risk-free interest rate), and the second – stochastic differential equation against GBM process, describing the current stock price.

Over time, algorithms for pricing financial options were transferred into a complex world of real investments. The first real option pricing models assumed – *per analogiam* – the existence of a single source of uncertainty which usually was the price of a product or commodity. The underlying asset was therefore modeled according to Geometric Brownian Motion. Over time, the idea of aggregate uncertainty emerged in the real options theory – it was considered that in the case of option valuation, the most convenient – from the perspective of the analyst – would be adopting the gross project value as the underlying asset. This assumption radically simplified the valuation process in the Real Options Analysis (ROA). However, despite the obvious fact that the project may take on negative values (occurrence of losses, where operating costs exceed price levels), ROA automatically adopted the GBM process. The purpose of this article is to revise validity of GBM process for real options and to propose a return to the Bachelier model. Advantages of the ABM process in terms

of the description of changes in the underlying asset in option pricing models have been raised so far by a small number of authors [e.g. Poitras, 1998; Trojanowska, Kort, 2005].

The article is organized as follows: Section 2 presents the concept of the stochastic process and Arithmetic and Geometric Brownian Motion, while section 3 – the valuation of an investment project with an option-to-temporarily-shut-down in multiplicative and additive binomial model.

## 1. Arithmetic vs. geometric stochastic process

The stochastic process is a description of changes in parameter value over time in a way that is at least partially random. The stochastic process can be:

- 1) continuous-time process where the time subscript  $t$  is a continuous variable – parameter changes occur at any time,
- 2) discrete-time process, where parameter changes occur at specific time intervals.

### 1.1. Brownian motion

The Wiener process variable  $z(t)$ , also called a Brownian motion, presents three essentials characteristics:

- 1) it is a Markov process, which means that the probability distribution of all possible future values of the process depends on the current value of the parameter only; this value is the only significant indicator of the future forecasting; the values of the past are irrelevant to the process (the process is memoryless);
- 2) changes in parameter ( $\Delta z$ ) at small time intervals are independent of each other, i.e. the probability distribution of parameter changes at any time interval is independent of another time interval (intervals do not overlap);
- 3) the distribution of parameter changes ( $\Delta z$ ) in any finite time interval is a normal distribution with mean  $E(\Delta z) = 0$  and variance growing linearly with time:  $\text{Var}(\Delta z) = \Delta t$ .

$$\Delta z = \varepsilon_t \sqrt{\Delta t} \tag{1}$$

where:

$\varepsilon_t$  – is a normally distributed random variable with a mean of 0 and a standard deviation of 1.

For the continuous time process ( $\Delta t \rightarrow 0$ ) the change of parameter in the standard Wiener process is:

$$dz = \varepsilon_t \sqrt{dt} \quad (2)$$

with expected value  $E(dz) = 0$  and variance  $\text{Var}(dz) = dt$ .

Brownian motion is a limit of a random walk. From the Central Limit Theorem results that the variance of the change in a Wiener process grows linearly with the time horizon.

### 1.1.1. Arithmetic Brownian Motion

The Wiener process can be generalized. A simple generalization of the process is Brownian motion with drift or Arithmetic Brownian Motion (ABM), in the form of:

$$dx = \alpha dt + \sigma dz \quad (3)$$

where:

- $dz$  – the increment of a Wiener process,
- $\alpha$  – the drift parameter,
- $\sigma$  – the variance.

It should be noted that over any time interval  $\Delta t$ , the change in  $x$  ( $\Delta x$ ) is normally distributed and has expected value  $E(\Delta x) = \alpha dt$  and variance  $\text{Var}(\Delta x) = \sigma^2 dt$ .

Figure 1 shows sample paths of changes in  $x$  (with trend line) in Brownian motion with drift where individual parameters are at levels:

- the initial value:  $x_0 = 10$ ,
- the drift parameter:  $\alpha = 20\%$ ,
- variability:  $\sigma = 25\%$ ,
- number of steps: 100.

### 1.1.2. Generalized Brownian motion – Itô processes

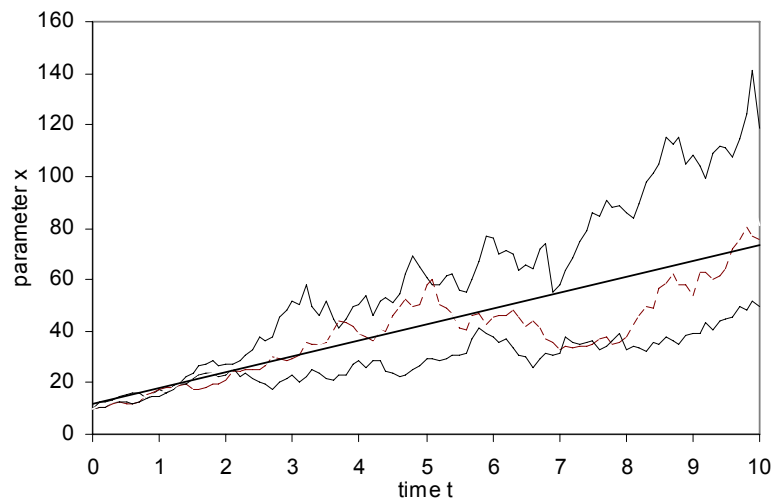
A generalized Brownian motion is a continuous-time stochastic process represented as:

$$dx = a(x, t)dt + b(x, t)dz \quad (4)$$

where:

$dz$  – the increment of a Wiener process,

$a(x, t)$ ,  $b(x, t)$  – known non-random functions of time and location.



**Figure 1.** Sample paths of Brownian motion with drift  
Source: Own study.

The generalized Brownian motion is called an Itô process. The expected continuous-time drift rate is determined by the formula  $E(dx) = a(x, t)dt$ , while variance – for  $dt$  infinitely small –  $\text{Var}(dx) = b^2(x, t)dt$  [Dixit, Pindyck, 1994].

### 1.2. Geometric Brownian Motion

In the practice of option pricing an essential role is played by a special case of the Itô process – Geometric Brownian Motion which occurs when  $a(x, t) = ax$  and  $b(x, t) = \sigma x$  (where  $a$  and  $\sigma$  are constants).

In this case the formula (4) becomes:

$$dx = \alpha x dt + \sigma x dz \quad (5)$$

As mentioned earlier, percentage changes in  $x$  ( $\Delta x/x$ ) are normally distributed. For the price, this requirement cannot be met – prices can never be negative. For this reason, it is assumed that price changes are log-normally distributed. Using Itô's lemma it can be shown that if  $x(t)$  is defined by the equation (5), then  $F(x) = \log x$  follows a simple Brownian motion with drift:

$$dF = \left(\alpha - \frac{1}{2}\sigma^2\right) dt + \sigma dz \quad (6)$$

Hence, over any finite time interval  $t$  the change in  $\log x$  is normally distributed with mean  $(\alpha - \frac{1}{2}\sigma^2)t$  and variance  $\sigma^2 t$ .

Given the above and assuming that  $x = S$ , we can write that:

$$\frac{dS}{S} = \alpha dt + \sigma dz \quad (7)$$

where:

$\alpha$  – (constant) instantaneous expected return on an asset,

$\sigma$  – (constant) instantaneous standard deviation of asset returns,

$dz$  – differential of a standard Wiener process (with mean 0 and variance  $dt$ ).

For discrete processes the equation (7) is:

$$\frac{\Delta S}{S} = \alpha \Delta t + \sigma \sqrt{\Delta t} \tilde{\varepsilon} \quad (8)$$

where:

$\Delta S$  – change in the asset price at a small time interval  $\Delta t$ ,

$\tilde{\varepsilon}$  – random sample from a standardized normal distribution,

$\alpha$  – expected (multiplicative) return on an asset per time unit,

$\sigma$  – volatility of returns on asset prices.

The expression  $\alpha \Delta t$  is a deterministic element, measuring the growth line angle, while the expression  $\sigma \sqrt{\Delta t} \tilde{\varepsilon}$  includes a stochastic element that determines the extent of fluctuations around the trend line.

Equation (5), called Geometric Brownian Motion (GBM) with drift or standard Wiener diffusion process, is used to model stock prices, interest rates, commodity prices, wage rates and other financial and economic variables.

## 2. Valuation of a project with an option to temporarily shut down production

A modern approach to the valuation of real assets allows for a departure from understanding the value through the prism of the classical NPV, now called as “static”, “passive” or “direct” value [Trigeorgis, 1996]. It allows for a more comprehensive assessment of investment opportunities by taking into account the value of managerial flexibility in relation to operational strategies, which is reflected in the concept of the so-called Expanded (or strategic) Net Present Value (XNPV), which is understood as the sum of the classical NPV and the option premium resulting from the value of managerial flexibility and different interactions between the different possibilities values of strategic dimension:

$$XNPV = NPV + OP \quad (9)$$

where:

$XNPV$  – strategic or expanded NPV,

$NPV$  – project’s value obtained in classical DCF analysis,

$OP$  – option premium.

Option value can be related both to net present and gross project value. In the latter case we can write:

$$XPV = PV + OP \quad (10)$$

where:

$XPV$  – strategic or expanded PV,

$PV$  – present gross project value.

### 2.1. Binomial options pricing model

The stochastic process approximation is represented by the binomial tree [Cox, Ross, Rubinstein 1979]:

- 1) multiplicative (Figure 2) – in the case of Geometric Brownian Motion,
- 2) additive (Figure 3) – in the case of Arithmetic Brownian Motion.

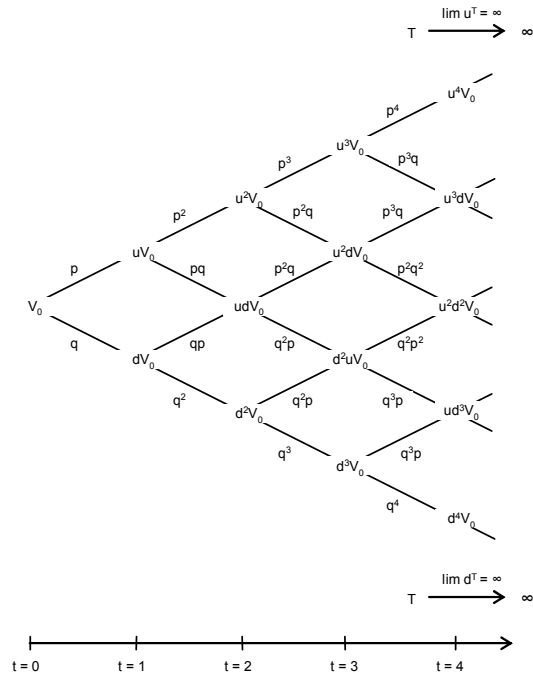


Figure 2. Multiplicative (geometric) stochastic process

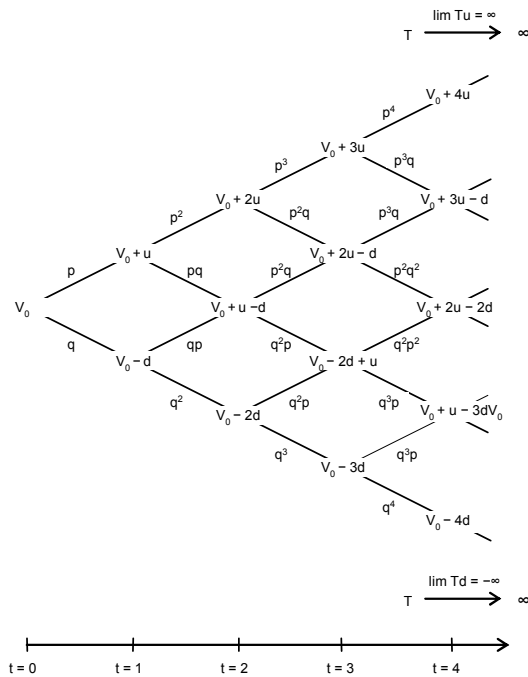


Figure 3. Additive (arithmetic) stochastic process



In the multiplicative process subsequent values of the asset – up and down movements – are results of the directly preceding value and multiplicative factors: up  $u$  ( $>1$ ) and down movements  $d$  ( $<1$ ); it is usually assumed that  $d = 1/u$ . Values in the extreme upper branch of the multiplicative tree tend to plus infinity and in the extreme lower branch – to zero.

On the other hand, in the additive process, two subsequent possible values of the process are:

- the sum of the preceding value and the up-movement factor  $u$ ,
- the difference between the preceding value and the down-movement factor  $d$ .

Values in extreme branches of the additive tree tend to:

- plus infinity – in uppermost part,
- minus infinity – in lowest part.

In the limit, as the number of steps tends to infinity, the end outcomes of the multiplicative tree approach a lognormal distribution, while in the additive tree – a normal distribution.

Options pricing in multiplicative binomial model can be realized by means of two approaches (Figure 4):

- replicating portfolio approach,
- risk-neutral probability approach.

Due to the fact that in the additive model the risk-neutral probability of up and down movements depends on  $V_0$ , it is impossible to apply the latter valuation approach in this model.

REPLICATING PORTFOLIO APPROACH		RISK-NEUTRAL PROBABILITY APPROACH	
Number of twin security units	$m = \frac{ROV_u - ROV_d}{V_u - V_d}$	Up and down movements factors:	$u = e^{\sigma\sqrt{\Delta t}}; d = \frac{1}{u}$
Number of 'risk-free' security units	$B = \frac{V_d ROV_u - V_u ROV_d}{(V_u - V_d)e^{r_f \Delta t}}$	Risk-neutral probability:	$p = \frac{e^{r_f \Delta t} - d}{u - d}$
Value of project with flexibility	$ROV_0 = mV_0 - B$	Value of project with flexibility	$ROV_0 = \frac{p(ROV_u) + (1-p)ROV_d}{e^{r_f \Delta t}}$

**Figure 4.** The solutions of the Cox-Ross-Rubinstein multiplicative binomial tree model  
Source: Own study.

## 2.2. Valuation of a project with an option to-temporarily-shut-down

The differences between valuation algorithms in multiplicative and additive models are shown on the example of a project valuation with an option-to-temporarily-shut-down current operations.

The conventional DCF analysis assumes that in each year the project will generate cash flows ensuring its performance. In reality, however, the manager has the flexibility to shut down operations temporarily – such action may be justified when the amount of revenue is lower than operating costs. The implementation of such options as option-to-temporarily-shut-down or option-to-abandon may significantly reduce potential losses of shareholders.

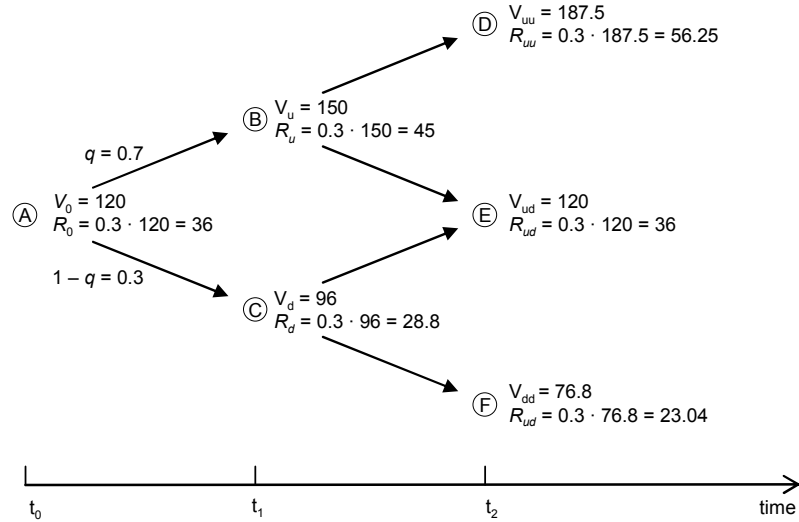
Options-to-temporarily-shut-down frequently appear in mining and exploration projects. For example: the management of a polymetallic mine, at any time of the project has the option – the right but not the obligation – to shut down operations when prices fall below the mine's unit operating costs and there is no hope of turning the tables on it soon.

### 2.2.1. Valuation of a project in multiplicative binomial model

Due to the fact that in the multiplicative binomial model the value of the project cannot take on negative values, production may be viewed as a call option on revenues in a given year with the exercise price which are operating variable costs ( $K_V$ ).

Suppose that the underlying asset following the stochastic process shown in Figure 2, provides in a given year revenues of 30 percent of the project value,  $R = 0.3V$  (Figure 5). In order to obtain these revenues the management pays \$18.1 million of operating variable costs per year and \$7 million of fixed costs. Assume that the low efficiency of the production process may cause higher variable costs to a level of \$30 million, but as long as the amount of revenues  $R$  exceeds the level of these costs, the production may still be profitable. This occurs when market conditions turn favorable. If these conditions get worse, the only appropriate action may be to stop operations temporarily. The option-to-temporarily-shut-down production at any time gives the management the right to obtain the value  $V$  of the underlying asset in return for paying the lesser of two values:

- variable costs  $K_V$ , when the project develops successfully or
- revenues  $R$ , which would be lost if, in the event of adverse changes in the market production was continued.



**Figure 5.** An underlying asset and cash revenues multiplicative tree of the project with option-to-temporarily-shut-down (cash values in Mill US\$)  
Source: Own study.

Denominating fixed costs by  $k_F$ , the payout for the project ( $ROV$ ) in final nodes is determined by the formula:

$$ROV = (V - k_F) - \min(K_V, R) \text{ or}$$

$$ROV = \max(V - K_V - k_F; V - R - k_F).$$

It should be noted that the increasing of variables costs up to \$30 million has as a result the decreasing of project present value from \$120 to \$ 86.44 million:

$$[p^2 \cdot 0.3 \cdot 187.5 + 2p(1 - p) \cdot 0.3 \cdot 120 + (1 - p)^2 \cdot 0.3 \cdot 76.8] / (1 + 0.05)^2 = 86.44$$

where:

$p$  – risk-neutral probability ( $p = 0.56$ )

Table 1 shows payouts for project in final nodes. Valuation of the project is presented in Figure 6 and in Table 2.

Table 1

Payouts for both project with flexibility and option-to-temporarily-shut-down in the multiplicative binomial model

node	Project payout (ROV) [Mill US\$]	Value of project without flexibility [Mill US\$]	Option payout (OP) [Mill US\$]
D	$\max[(V_{uu} - K_V - k_F), (V_{uu} - 0.3V_{uu} - k_F)] = \max[(187.5 - 30 - 7), (187.5 - 0.3 \cdot 187.5 - 7)] = 150.5$	$V_{uu} - K_V - k_F = 150.5$	$\max[0, (V_{uu} - 0.3V_{uu} - k_F) - (V_{uu} - K_V - k_F)] = \max(0, 124.25 - 150.5) = 0$
E	$\max[(V_{ud} - K_V - k_F), (V_{ud} - 0.3V_{ud} - k_F)] = \max[(120 - 30 - 7), (120 - 0.3 \cdot 120 - 7)] = 83$	$V_{ud} - K_V - k_F = 83$	$\max[0, (V_{ud} - 0.3V_{ud} - k_F) - (V_{ud} - K_V - k_F)] = \max(0, 77 - 83) = 0$
F	$\max[(V_{dd} - K_V - k_F), (V_{dd} - 0.3V_{dd} - k_F)] = \max[(76.8 - 30 - 7), (76.8 - 0.3 \cdot 76.8 - 7)] = 46.76$	$V_{dd} - K_V - k_F = 39.8$	$\max[0, (V_{dd} - 0.3V_{dd} - k_F) - (V_{dd} - K_V - k_F)] = \max(0, 46.76 - 39.8) = 6.96$

Source: Own study.

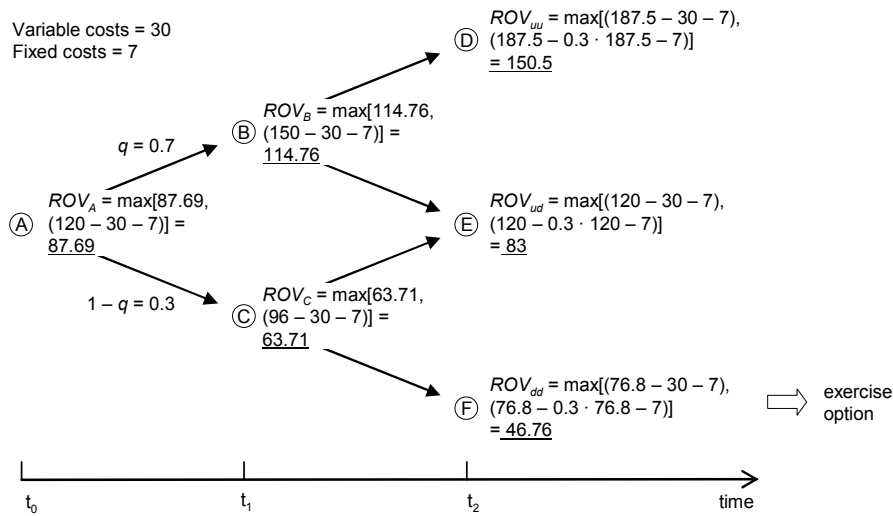


Figure 6. Valuation of project (XPV value) with option-to-temporarily-shut-down in the multiplicative binomial model (cash values in Mill US\$)

Source: Own study.

The results show that the Expanded Present Value (XPV) stands at \$87.69 million and exceeds the PV (\$86.44 million) by managerial flexibility value (OP = \$1.25 million).

**2.2.2. Valuation of a project in additive binomial model**

In the case when the underlying asset (gross project value) is modeled with the additive binomial tree another approach to value the option-to-temporarily-shut-down can be shown.

The main effect of shutting down production temporarily is variable cost saving. It should be noted that in the case of exercising this option certain fixed costs related to the shutdown of production may be incurred – in the case of mineral projects concerned basically with maintaining mine infrastructure (ventilation, dewatering, maintenance of workings), ensuring safety etc. Main shares of option exercise price are dismissal wages and maintenance costs of closed mine faces, ventilation, machines, equipment, etc.

As mentioned earlier, the value of the project under the additive model may take on negative values. Figure 7 shows changes in the value of the underlying asset with the present value  $V_0 = \$40$  million in two-period additive model with assumption that up and down parameters are the same ( $u = d = \$30$  million). The mine management has the option-to-shut-down operations temporarily when the project generates losses. That is the case in point „F” of the presented binomial tree (Figure 7) – the value of the underlying asset  $V_{dd}$  amounts to (\$20) million.

Suppose that the current operating costs of the studied project stand at \$30 million, of which fixed costs  $k_F$  are \$9 million and variable costs –  $k_V = \$21$  million. The costs of shutting down operations temporarily  $K_T$  were estimated at \$5 million. Deciding to shut down production temporarily the company will incur a cost of \$14 million ( $k_s' + K_T'$ , where index „prim” denotes the time-value adjustment – the „risk-free” rate was adopted at 5% level [Saługa, 2009]. In this situation, the profit of the company will be saving losses that would be incurred if the production was maintained:  $V - k_F' - K_T'$ . Thus, the positive effect of shutting down operations temporarily makes sense only if the absolute value of the total amount of shutting down and fixed costs gets into the range (0;  $V$ ) – otherwise rational action is to abandon operations.

Table 2

Estimation of expanded PV (XPV) and managerial flexibility (OP) values for project with option-to-temporarily-shut-down – a replicating portfolio approach in the multiplicative binomial model (cash values in Mill US\$)

Option parameters:	Process parameters:
1) Underlying asset: gross project value $V$ 2) Present value of underlying asset: $V_0 = 120$	1) Character of process: multiplicative (geometric)

Table 2 cont.

Option parameters:	Process parameters:
3) Type of option: american call option 4) Project revenues: $0.3V$ 5) Fixed costs: $k_F = 7$ 6) Exercise price (nominal): $K_V = 30$ 7) Lifetime of option: $\tau = 2$ years 8) Modification of underlying asset: $V - 0.3V - K_F$	2) Up-movement parameter: $u = 1.25$ 3) Down-movement parameter: $d = 1/u = 0.625$ 4) Objective probability of up-movement: $q = 0.7$ 5) Objective probability of down-movement: $1 - q = 0.3$ 6) Volatility: $\sigma = 22.31\%$ 7) „Risk-free” interest rate: $r_f = 5\%$
Value of project with flexibility (XPV)	Flexibility value (OP)
<p><b>a) node B:</b></p> $m \cdot V_{uu} - (1 + r_f)B = ROV_{uu} = \max[(187.5 - 30 - 7), (187.5 - 0.3 \cdot 187.5 - 7)]$ $m \cdot V_{ud} - (1 + r_f)B = ROV_{ud} = \max[(120 - 30 - 7), (120 - 0.3 \cdot 120 - 7)]$ $m \cdot 187.50 - (1 + 0.05)B = 150.5$ $m \cdot 120 - (1 + 0.05)B = 83$ $m = 1, B = 35.24$ $ROV_u = m \cdot V_u - B = 1 \cdot 150 - 35.24 = 114.76$ $ROV_B = \max[114.76, 150 - 0.3 \cdot 96 - 7] = \max(114.76, 98) = \underline{114.76}$ <p><b>b) node C:</b></p> $m \cdot V_{ud} - (1 + r_f)B = ROV_{ud} = \max[(120 - 30 - 7), (120 - 0.3 \cdot 120 - 7)]$ $m \cdot V_{dd} - (1 + r_f)B = ROV_{dd} = \max[(76.8 - 30 - 7), (76.8 - 0.3 \cdot 76.8 - 7)]$ $m \cdot 120 - (1 + 0.05)B = 83$ $m \cdot 76.80 - (1 + 0.05)B = 46.76$ $m = 0.84, B = 16.83$ $ROV_d = m \cdot V_d - B = 0.84 \cdot 96 - 16.83 = 63.71$ $ROV_C = \max(63.71, 96 - 0.3 \cdot 96 - 7) = \underline{63.71}$ <p><b>c) node A:</b></p> $m \cdot V_u - (1 + r_f)B = ROV_B = 114.76$ $m \cdot V_d - (1 + r_f)B = ROV_C = 63.71$ $m \cdot 150 - (1 + 0.05)B = 114.76$ $m \cdot 96 - (1 + 0.05)B = 63.71$ $m = 0.95, B = 25.77$ $ROV_0 = m \cdot V_0 - B = 0.95 \cdot 120 - 25.77 = 87.69$ $XPV = ROV_A = \max(87.69, 120 - 0.3 \cdot 120 - 7) = \underline{87.69}$	<p><b>a) node B:</b></p> $m \cdot V_{uu} - (1 + r_f)B = OP_{uu} = \max[(187.5 - 0.3 \cdot 187.5 - 7) - (187.5 - 30 - 7), 0] = \max(-26.25, 0)$ $m \cdot V_{ud} - (1 + r_f)B = OP_{ud} = \max[(120 - 0.3 \cdot 120 - 7) - (120 - 30 - 7)] = \max(-6.25, 0)$ $m \cdot 187.50 - (1 + 0.05)B = 0$ $m \cdot 120 - (1 + 0.05)B = 0$ $m = 0, B = 0$ $OP_u = OP_B = \underline{0}$ <p><b>b) node C:</b></p> $m \cdot V_{ud} - (1 + r_f)B = OP_{ud} = \max[(120 - 0.3 \cdot 120 - 7) - (120 - 30 - 7)] = \max(-6.25, 0)$ $m \cdot V_{dd} - (1 + r_f)B = OP_{dd} = \max[(76.8 - 0.3 \cdot 76.8 - 7) - (76.8 - 30 - 7)] = \max(6.96, 0)$ $m \cdot 120 - (1 + 0.05)B = 0$ $m \cdot 76.80 - (1 + 0.05)B = 6.96$ $m = -0.16, B = -18.4$ $OP_d = -0.16 \cdot 96 + 18.4 = 2.95$ $OP_C = \max[2.95, (96 - 0.3 \cdot 96 - 7) - (96 - 30 - 7)] = \max(2.95, -27) = \underline{2.95}$ <p><b>c) node A:</b></p> $m \cdot V_u - (1 + r_f)B = OP_B = 0$ $m \cdot V_d - (1 + r_f)B = OP_C = 2.95$ $m \cdot 150 - (1 + 0.05)B = 0$ $m \cdot 96 - (1 + 0.05)B = 2.95$ $m = -0.05, B = -7.79$ $OP_0 = -0.05 \cdot 120 + 7.79 = 1.25$ $OP_A = \max[1.25, (120 - 0.3 \cdot 120 - 7) - (120 - 30 - 7)] = \max(1.25, -42) = \underline{1.25}$

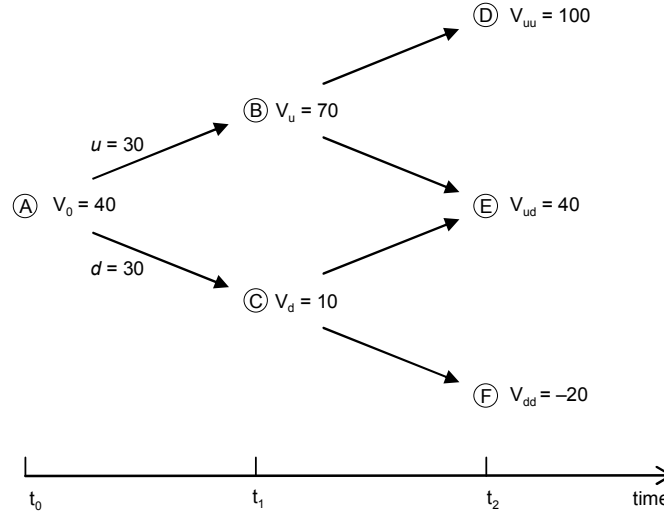
Source: Own study.

Table 3

Estimation of expanded PV (XPV) and managerial flexibility (OP) values for project with option-to-temporarily-shut-down – a risk-neutral probability approach in the multiplicative binomial model (cash values in Mill US\$)

Option parameters:	Process parameters:
1) Underlying asset: gross project value $V$ 2) Present value of underlying asset: $V_0 = 120$ 3) Type of option: american call option 4) Project revenues: $0.3V$ 5) Fixed costs: $k_F = 7$ 6) Exercise price (nominal): $K_V = 30$ 7) Lifetime of option: $\tau = 2$ years 8) Modification of underlying asset: $V - 0.3V - k_F$	1) Character of process: multiplicative (geometric) 2) Up movement parameter: $u = 1.25$ 3) Down movement parameter: $d = 1/u = 0.625$ 4) Objective probability of up-movement: $q = 0.7$ 5) Objective probability of down-movement: $1 - q = 0.3$ 6) Volatility: $\sigma = 22.31\%$ 7) „Risk-free” interest rate: $r_f = 5\%$
Value of project with flexibility (XPV)	Flexibility value (OP)
$p = (1 + r_f - d)/(u - d) = (1.05 - 0.625)/(1.25 - 0.625) = 0.56$ <p>a) node B:</p> $ROV_u = (p \cdot ROV_{uu} + (1 - p)ROV_{ud})/(1 + r_f) = (0.56 \cdot 150.5 + 0.44 \cdot 83)/1.05 = 114.76$ $ROV_B = \max(114.76, 150 - 0.3 \cdot 96 - 7) = 114.76$ <p>b) node C:</p> $ROV_d = (p \cdot ROV_{ud} + (1 - p)ROV_{dd})/(1 + r_f) = (0.56 \cdot 98 + 0.44 \cdot 46.76)/1.05 = 63.71$ $ROV_C = \max(63.71, 96 - 0.3 \cdot 96 - 7) = 63.71$ <p>c) node A:</p> $ROV_0 = (p \cdot ROV_B + (1 - p)ROV_C)/(1 + r_f) = (0.56 \cdot 114.76 + 0.44 \cdot 63.71)/1.05 = 87.69$ $ROV_A = \max(87.69, 120 - 0.3 \cdot 120 - 7) = 3.71$	$p = (1 + r_f - d)/(u - d) = (1.05 - 0.625)/(1.25 - 0.625) = 0.56$ <p>a) node B:</p> $OP_u = (p \cdot OP_{uu} + (1 - p)OP_{ud})/(1 + r_f) = (0.56 \cdot 0 + 0.44 \cdot 0)/1.05 = 0$ $OP_B = OP_u = 0$ <p>b) node C:</p> $OP_d = (p \cdot OP_{ud} + (1 - p)OP_{dd})/(1 + r_f) = (0.56 \cdot 0 + 0.44 \cdot 6.96)/1.05 = 2.95$ $OP_C = \max[2.95, (96 - 0.3 \cdot 96 - 7) - (96 - 30 - 7)] = 2.95$ <p>c) node A:</p> $OP_0 = (p \cdot OP_B + (1 - p)OP_C)/(1 + r_f) = (0.56 \cdot 0 + 0.44 \cdot 2.95)/1.05 = 1.25$ $OP_A = \max[1.25, (120 - 0.3 \cdot 120 - 7) - (120 - 30 - 7)] = 1.25$

Source: Own study.



**Figure 7.** An underlying asset additive tree of the project with option-to-temporarily-shut-down (values in Mill US\$)

Source: Own study.

Table 3 shows the payouts for a project in final nodes, Figure 8 and Table 4 – valuation of the project (including valuation of flexibility) in the replicating portfolio approach.

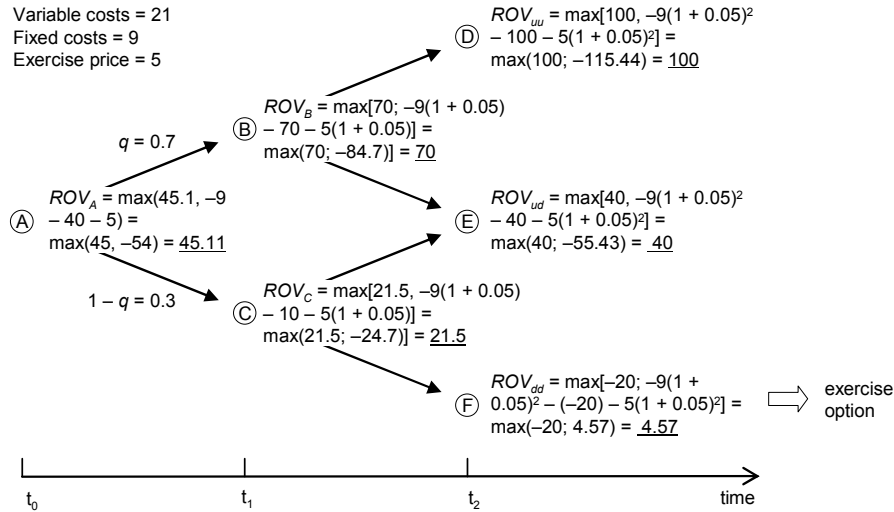
Table 4

Payouts for both project with flexibility and option-to-temporarily-shut-down in the additive binomial tree approach

node	Project payout (ROV) [Mill US\$]	Value of project without flexibility [Mill US\$]	Option payout (OP) [Mill US\$]
D	$\max[V_{uu}; -k_F(1 + 0.05)^2 - V_{uu} - K_T(1 + 0.05)^2] = \max(100; -115.44) = 100$	100	0
E	$\max[V_{ud}; -k_F(1 + 0.05)^2 - V_{ud} - K_T(1 + 0.05)^2] = \max(40; -55.43) = 40$	40	0
F	$\max[V_{dd}; -k_F(1 + 0.05)^2 - V_{dd} - K_T(1 + 0.05)^2] = \max(-20; 4.57) = 4.57$	-20	$ROV_{dd} - V_{dd} = 4.57 - (-20) = 24.57$

Source: Own study.





**Figure 8.** Valuation of project (XPV value) with option-to-temporarily-shut-down in the additive binomial tree approach (cash values in Mill US\$)  
 Source: Own study.

Calculations show that the value of the studied project with an option-to-temporarily-shut-down production, that is the expanded present value (XPV), stands at \$45.11 million; the value of flexibility related to the opportunity to shut down operations temporarily (option premium – OP) amounts to \$5.11 million.

## Conclusions

The standard model describing a behavior of the underlying security (which typically is a gross project value) in the real options analysis is Geometric Brownian Motion (GBM). This process has been directly adopted from financial options pricing models. In the GBM model end values of the stochastic process are log-normally distributed. The GBM assumption makes the ROA valuation unrealistic because in real situations project’s cash flows can be negative. Classical cases of negative cash flows usually appear in situations where production operating costs exceed the level of prices. In such cases managers consider the possibility of shutting down production temporarily and even abandoning the project. According to the author’s opinion, in the real options analysis, a more appropriate model for describing the behavior of the underlying asset is the Arithmetic Brownian Motion (ABM), under which the values can go negative. This means returning to the beginnings of the theory of options [Bachelier, 1900].

Table 5

Estimation of expanded PV (XPV) and managerial flexibility (OP) values for project with option-to-temporarily-shut-down – a replicating portfolio approach in the additive binomial model (cash values in Mill US\$)

Option parameters:	Process parameters:
1) Underlying asset: gross project value $V$ 2) Present value of underlying asset: $V_0 = 40$ 3) Type of option: american call option 4) Current exercise price: $K_T = 5$ 5) Fixed costs: $k_F = 9$ 6) Variable costs: $k_V = 21$ 7) Lifetime of option: $\tau = 2$ years 8) Modification of underlying asset: $V - k_F^2 - K_T^2$	1) Character of process: additive (arithmetic) 2) Up-movement parameter: $u = 30$ 3) Down-movement parameter: $d = 30$ 4) „Risk-free” interest rate: $r_f = 5\%$
Value of project with flexibility (XPV)	Flexibility value (OP)
<p><b>a) node B:</b></p> $m \cdot V_{uu} - (1 + r_f)B = ROV_{uu} = \max[100, -9(1 + 0.05)^2 - 100 - 5(1 + 0.05)^2] = 100$ $m \cdot V_{ud} - (1 + r_f)B = ROV_{ud} = \max[40, -9(1 + 0.05)^2 - 40 - 5(1 + 0.05)^2] = 40$ $m = 1, B = 0$ $ROV_u = m \cdot V_u - B = 1 \cdot 70 - 0 = 70$ $ROV_B = \max[70, -9(1 + 0.05) - 70 - 5(1 + 0.05)] = \underline{70}$ <p><b>b) node C:</b></p> $m \cdot V_{ud} - (1 + r_f)B = ROV_{ud} = \max[40, -9(1 + 0.05)^2 - 40 - 5(1 + 0.05)^2] = 40$ $m \cdot V_{dd} - (1 + r_f)B = ROV_{dd} = \max[-20, -9(1 + 0.05)^2 - (-20) - 5(1 + 0.05)^2] = 4.57$ $m = 0.59, B = -15.6$ $ROV_d = m \cdot V_d - B = 0.59 \cdot 10 + 15.6 = 21.5$ $ROV_C = \max(21.5, -9(1 + 0.05) - 10 - 5(1 + 0.05)) = \underline{21.5}$ <p><b>c) node A:</b></p> $m \cdot V_u - (1 + r_f)B = ROV_B = 70$ $m \cdot V_d - (1 + r_f)B = ROV_C = 21.5$ $m = 0.81, B = -12.78$ $ROV_0 = m \cdot V_0 - B = 0.81 \cdot 40 + 12.78 = 45.11$ $XPV = ROV_A = \max(45.11, -9 - 40 - 5) = \underline{45.11}$	<p><b>a) node B:</b></p> $m \cdot V_{uu} - (1 + r_f)B = OP_{uu} = 0$ $m \cdot V_{ud} - (1 + r_f)B = OP_{ud} = 0$ $m = 0, B = 0$ $OP_u = OP_B = \underline{0}$ <p><b>b) node C:</b></p> $m \cdot V_{ud} - (1 + r_f)B = OP_{ud} = 0$ $m \cdot V_{dd} - (1 + r_f)B = OP_{dd} = 4.57 - (-20) = 24.57$ $m \cdot 120 - (1 + 0.05)B = 0$ $m \cdot 76.80 - (1 + 0.05)B = 6.96$ $m = -0.41, B = -15.6$ $OP_d = -0.41 \cdot (-20) + 15.6 = 11.5$ $OP_C = \max[11.5, -9(1 + 0.05) - 10 - 5(1 + 0.05) - 10] = \max(11.5, -34.7) = \underline{11.5}$ <p><b>c) node A:</b></p> $m \cdot V_u - (1 + r_f)B = OP_B = 0$ $m \cdot V_d - (1 + r_f)B = OP_C = 11.5$ $m \cdot 70 - (1 + 0.05)B = 0$ $m \cdot 10 - (1 + 0.05)B = 11.5$ $m = -0.19, B = -12.78$ $OP_0 = -0.19 \cdot 40 + 12.78 = 5.11$ $OP_A = \max(5.11, -9 - 40 - 5 - 40) = \max(5.11, -94) = \underline{5.11}$

Source: Own study.

The difference between GBM and ABM approaches in the real options valuation has been presented on the example of an option-to-temporarily-shut-down production which is typically available for owners of marginal properties, who are flexible in shutting down and restarting operations. Exercising this option management saves variable costs that would be incurred through the implementation of unprofitable production. In the valuation model which assumes that changes in the underlying asset follow multiplicative binomial model the option-to-shut-down is a call option on revenues in a given year with an exercise price that are operational variable costs – the pricing algorithm becomes complicated, however, when the shutting down costs are being incorporated into the model. On the other hand the valuation process becomes essentially simplified if applying the additive tree – the costs of temporarily closed operations are incorporated directly into the option pricing algorithm as the exercise price of an American option-to-temporarily-shut-down.

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