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A key factor of the DCF model coherency

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Abstract

Aim/purpose – The aim of this paper is to provide economically justified evidence that the business value calculated by income valuation methods is the same, regardless of the type of cash flow used in the valuation algorithm.

Design/methodology/approach – The evidence was arrived at using free cash flow to equity (FCFE), debt (FCFD) and firm (FCFF). The article draws attention to the FCFF method's particular popularity in income valuation, based on analysts' practice. It shows an overview of various approaches to determine the capital structure in the formula for WACC, both in practice and theory. Finally, it examines an empirical example with the authors' own derivations and postulates.

Findings – The conclusion drawn from the conducted analysis is that the key to the reconciliation process, and thus DCF model coherency, is to apply the appropriate method of capital structure estimation during the calculation of the weighted average cost of capital (WACC). This capital structure will henceforth be referred to as 'income weights'.

Research implications/limitations – It should be noted that the obtained compliance of valuation results does not imply that the income valuation becomes an objective way of determining business value. It still remains subjective.

Originality/value/contribution – According to the presented approach, the DCF model's subjectivism is limited to the forecasts. The rest is the algorithm which, based on the principles of mathematics, should be used in the same way in every situation.

Keywords: income valuation, DCF, weighted average cost of capital, WACC, reconciliation.

JEL Classification: G32, M21, C67, G12, C02.

1. Introduction

The concepts of value and valuation occupy a special place in the theory and practice of hundreds of thousands of financial analysts. This is because they provide a way to answer the question how many units of the currency, in theory, an audited company is worth.

The most popular valuation approach – income based¹ – is inextricably linked with the concept of cost of capital, which since at least the sixties of the twentieth century has caused theorists to reflect deeply and practitioners to be vexed [Modigliani & Miller 1958, 1963]. It seems that the idea of the weighted average cost of capital is simple to use and known to all the scientific and professional environments. However, in light of the literature, we can conclude that in this area opponents of alternative individual technical nuances may run a tilt for years [Pęksyk 2012].

The literature presents a variety of approaches towards the estimation of the future capital structure in the WACC equation of companies. At the same time, among all of these approaches, the literature in the field does not prove its own thesis of mutual compatibility of the various DCF valuation methods [Copeland, Koller & Murrin 1990; Damodaran 1994; Benninga & Sarig 1997]. Due to this fact, it is still an open question whether the DCF model is coherent and how to prove this. In this area, the authors have found a research gap and stated the objective of the present paper to examine the conditions of the claimed compatibility. Having conducted research, the authors provided the proof of mutual compatibility of the various DCF valuation methods with a detailed explanation and proposed a reconciliation process that results in a coherent DCF model.

The article consists of five main sections. The first one is an introduction, which describes the problem and the papers' objectives. The second section consists of a literature and practice review, which presents both the theoretical background and statistics of DCF model usage by financial experts. The third section presents the research methodology, which describes the study method and its stages. The fourth section shows the research findings, which presents evidence for the economic reasonable equivalence of DCF model's methods in the approach of income weights in WACC. The last section contains the authors' conclusions, which show the contribution of findings and limitations of the coherent DCF model.

¹ When the 'income valuation' method uses discounted cash flow as an income, then it is called 'DCF valuation', and because this paper focuses directly on cash flows in their three forms, these names are used interchangeably.

2. Literature and practice review

This section provides two sides of existing and relevant knowledge of the mutual compatibility of the various DCF valuation methods.

On the one hand, it presents a thesis of coherence in the DCF model in the literature and goes deeper to show the existence of a variety of possibilities to estimate the companies' capital structure. These approaches are shown with reference to the particular literature and compared to each other.

On the other hand, it reviews the existing research on the practice of using the DCF model among financial industry experts. It also presents the authors' study in the form of statistics, which are based on the literature.

Results show that theorists are not unanimous, and practitioners, based on their knowledge and experience, enjoy a whole range of possibilities given by the theory.

2.1. Theories of estimating the company's future capital structure in the context of a coherent DCF model

There are two approaches to estimating equity value using DCF valuation: direct and sequential [Benninga & Sarig 1997], as depicted in Figure 1. The first one, Free Cash Flow to Equity (FCFE) uses cost of equity (r_e) as a discount rate, so that after adjusting for non-operating assets (NOA), it yields results in the value of the company's equity. The second process uses Free Cash Flows to Firm (FCFF) as income, and it discounts them by the weighted average cost of capital (WACC), so that after adjusting for non-operating assets, it results in the value of the whole company, in other words enterprise. In order to obtain the equity value, the debt or net debt² must be deducted.

If the discount rates are appropriately selected and properly reflect the risks associated with the chosen cash flow, the equity value calculated using a model based on the total discounted cash flows will be the same as obtained by discounting the direct cash flows for shareholders [Copeland, Koller & Murrin 1990; Damodaran 1994].

² If it is established that the non-operating assets include cash, then debt, rather than net debt, should be subtracted. This is because otherwise cash held by the enterprise would be included in the measurement twice: once during the addition of non-operating assets and once during the deduction of net debt from enterprise value.

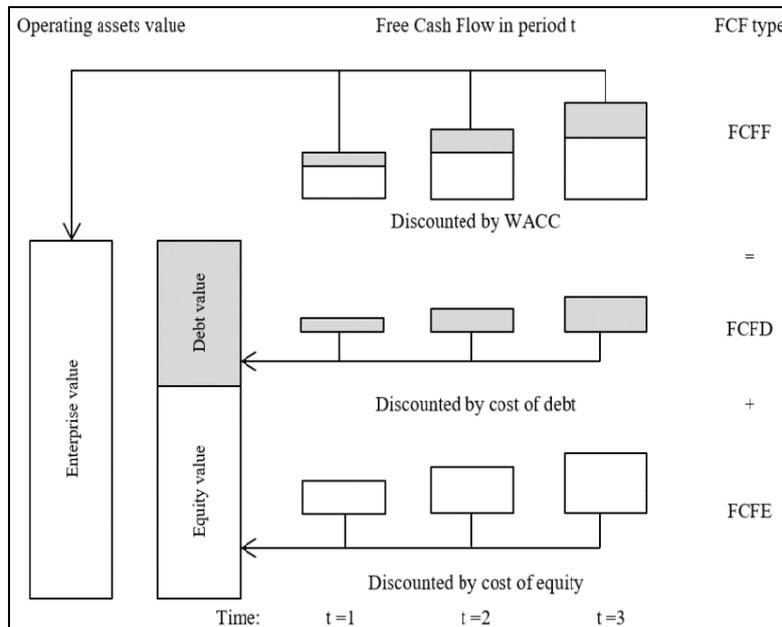
The literature indicates that both of the DCF valuation approaches should be equivalent (Figure 2), but in the area of estimating WACC the literature presents analysts with a wide array of possibilities, which often arouse much controversy. With stringent assumptions, both procedures should lead to identical results. However, in practice, these assumptions are not met, and the direct and sequential valuation processes give different results [Benninga & Sarig 1997, pp. 88].

Figure 1. Direct and sequential ways of estimating the equity value using DCF valuation

Type of operating income	for owners	for capital providers
Discount rate	r_e	WACC
	+ value of non-operating assets	
	=	=
Valued capital	Equity	Enterprise - (Net) Debt
		=
Valued capital		Equity

Source: Based on Kuczowic [2012].

Figure 2. Estimating a company's operating assets value using DCF valuation



Source: Based on Copeland, Koller & Murrin [1990].

The controversy, in particular, relates to the correct way of estimating the future capital structure of a company³ whose knowledge is absolutely required for the calculation of WACC. Regardless of how r_e and r_d (cost of debt) will be evaluated, the formula for WACC⁴ is presented as follows:

$$WACC = w_e * r_e + w_d * r_d * (1 - t), \quad (1)$$

where:

w_e – weight of equity value,

w_d – weight of debt value. Wherein:

$$w_e + w_d = 1. \quad (2)$$

The literature⁵ shows different approaches to determining the weight of equity and weight of debt in the formula for WACC. In particular, the following should be mentioned:

1. Determination of the weights on the basis of market data.
2. Determination of the weights on the basis of book data.
3. Determination of the weights on the basis of a company's target capital structure.

Among the literature, market weights approach is the preferred method of perceiving the future capital structure [Kuczowic 2014]. It is postulated that the capital structure should be the way it is currently valued by the market. However, it was noted that this method contains a problem of a vicious circle. Its essence lies in the fact that in order to estimate the value of equity using the FCFF method, the WACC should be known; to determine the WACC, the capital structure should be known; in turn, to estimate the capital structure, the value of equity and debt should be known [Copeland, Koller & Murin 1990]. This is so because it was considered that:

³ In addition to the issue of estimating the capital structure, adjustments to WACC should be attended to. Research conducted among British analysts suggests significant differences between theory and practice [Arnold & Hatzopoulos 2000]. Typical statements from surveyed analysts were: “We use the formula of WACC plus a margin of safety; use the formula of WACC plus inflation; in fact, we use the average from the WACC and the lowest rate of return required by investors” [Arnold & Hatzopoulos 2000, pp. 619].

⁴ The special role of WACC calculation in the valuation of businesses, points out Wiśniewski [2008]. In his paper, he stated that “[...] any comments concerning the accuracy or mistakes in estimating the cost of capital made in this article shall not apply in other areas of the use of cost of capital, especially in the valuation of companies by the discounted cash flow method; as in the valuation, the cost of capital has a direct impact on the estimated value” [Wiśniewski 2008, pp. 115].

⁵ Among others, these methods are counted and widely described by Cornell [1993] and Kuczowic [2014].

$$w_e = \frac{MV_E}{MV_E + MV_D} \quad (3)$$

$$w_d = \frac{MV_D}{MV_E + MV_D} \quad (4)$$

where:

MV_E – Market Value of Equity,

MV_D – Market Value of Debt.

As a remedy to this problem, two solutions have been proposed. The first one is to estimate the weights of the individual capitals by assuming that the valued company in the long term will have the same structure as the peer group. One way of validating this postulate is to find a market weight of listed comparable companies, averaging them and adopting the results obtained in this way as reliable for the purpose of company valuation [Hitchner 2006; Pęksyk 2012].

Another proposed solution to the problem of market weights is to use the iterative method of determining the cost of capital. Literature contains broad empirical examples in the application of this concept [Mohanty 2007; Pratt & Grabowski 2008; Larkin 2011]. This method, in an iterative manner, determines the company's capital weights until it becomes a true equation:

$$V_E + V_D = V \quad (5)$$

where:

V – enterprise value.

However, it should be noted that the fixed WACC rate, which was obtained in an iterative process, is not economically justified, but only set at a level which ensures the compatibility of equation (5). This approach has been the subject of serious criticism for wishing to bring equality 'by force' [Matschke & Brösel 2011].

Taking into consideration the description of the market weights problem and postulated solutions, one reaches the conclusion that the market weights approach treats the intrinsic value (which is the result of the income valuation) and the market value in the same manner, between which there may not necessarily be an equals sign.

Although in the literature the market weights method is preferred, in a situation with difficulties in estimation, especially in the case of non-listed companies, the authors conditionally allow the application of the book weights [Cornell 1993; Mills 1998; Bartoszewicz, Pniewski & Szablewski (ed.) 2008]. The use of book weights is present in both the theory and practice of business valuation. Kuczowic [2012] does not agree with this approach, even in the case of small

and non-listed companies and proposes the previously mentioned market weight method, which should be estimated using the iterative process or a target capital structure method. Malinowska [2001] notes that estimating weights using the book method should be approached with caution. The main controversy of using this method is associated with the fact that it does not reflect the real economic claims of each capital provider, but only the value invested in the company by investors in the past [Copeland, Koller & Murrin 1990; Dudycz 2005; Grzywacz 2012].

Historically, the earliest concept is to determine the target capital structure, which the management of the company will seek and in the long term maintain. It assumes that the target capital structure should be optimal, that is, one which maximizes the value of the company [Modigliani & Miller 1958, 1963; Rappaport 1986; Rao & Stevens 2007]. It is worth noting that the correct application of the postulate of the target capital structure is coherent with respect to the DCF model. The recursive procedure for estimating the value of the company, starting from the assumption of a long-term financial plan imposed by the company's management, provides complete correctness of equation (5) [Cegłowski & Podgorski 2012], whereas the simulation analysis is able to indicate the level of optimal capital structure.

Research based on surveys conducted among 356 financial experts with CFA or equivalent designations, made across 10 European countries, sheds some light on applied approaches for estimating the future capital structure, in the process of the company valuation [Bancel & Mittoo 2014]. It showed that approximately⁶:

1. 46% surveyed analysts use the target market weights.
2. 34% surveyed analysts use the book weights.
3. 31% surveyed analysts use average sector weights.

For the authors, the high rate (34%) of book values proved to be the most incomprehensible. In order to understand it, they quoted two interesting comments of their respondents: “[...] the book value is a proxy that is far from perfect, but that may not be more ‘false’ than other measures [...] considering the book value makes sense when the return on capital engaged is not far from the WACC because, in that case, the book value of the firm equals its market value” [Bancel & Mittoo 2014, pp. 110].

⁶ The results do not add up to 100% because respondents could choose more than one answer. This means that in one case an analyst could use the market weights and in another situation book weights.

2.2. The practice of using DCF valuation methods

It is not uncommon that the same company, at exactly the same time, using exactly the same cash flow in income valuation method is valued differently by different analysts. This can happen to such an extent that one analyst can say to buy, while the other would say to sell. This was evidenced by the two vastly different valuations of the company Bogdanka S.A. Both were released on August 27, 2016. According to the Polish Association of Individual Investors, DM BOS analysts recommended buying with a target price of PLN 65.90, while Haitong analysts recommended selling with a target price of PLN 38.18 [www 1]. Another example is the valuation of the company CD Projekt S.A. from June 11, 2015. According to the same source, the previous recommendation released by Trigon DM on April 21 had been 'buy' with a target price of PLN 24 per share. In a report dated June 10 (made public on the broad market on June 11, 2015), this institution substantially increased the valuation per share to a level of PLN 38. On the other hand, DM BZ WBK cut its recommendation from 'hold' to 'sell' with a valuation of PLN 23 per share [www 2].

Although the press releases did not indicate which method of valuation had been applied, the author's consultations with these brokerage houses revealed that in order to estimate the value of the company at least the income valuation was applied. It should be noted that the final valuations presented in the recommendations are significantly influenced by the adopted weights of individual valuations, e.g., comparative and income valuation⁷.

Table 1 presents the results of these valuations taking into account only the income valuation, which in every case used Free Cash Flow to Firm (FCFF) and the weighted average cost of capital (WACC) as the discount rate.

Table 1. Results comparison of the income valuations issued on the same day

Valued company	Recommendation issuance date	Institution and their DCF valuation result			
		Dom Maklerski BOŚ S.A.	Haitong Bank S.A.	Dom Maklerski Trigon S.A.	Dom Maklerski BZ WBK S.A.
Bogdanka S.A.	27 Aug, 2016	48.5	41.9	–	–
CD Projekt S.A.	11 June, 2015	–	–	38.0	23.0

Source: Dom Maklerski BOŚ S.A., Haitong Bank S.A., Dom Maklerski Trigon S.A., Dom Maklerski BZ WBK S.A.

⁷ The only exception is the recommendation issued by the Trigon Brokerage House, which used only the income method.

If the difference in obtained equity value is the result of the adoption of various forecasts or estimates of the discount rates, provided that they have been thoroughly justified, there is no controversy. This is because no one can predict the future; we can only try to do it based on knowledge and experience. If, however, this difference also results from a different interpretation of the DCF model algorithm, then the result certainly is fraught with a higher degree of subjectivity.

In the face of this issue particularly interesting are research studies that examined 224 stock recommendations from the 13 Polish brokerage houses, between January 1 and September 30, 2010 [Głębocki et al. 2011]. It was stated that:

1. Among 224 examined recommendations, up to 222 times (99.1%) income valuation was used, in which 188 times (84.7%) FCFF was used.
2. In every case (100%) FCFF was discounted by the weighted average cost of capital.
3. In most cases (68%) the variable discount rate was used.
4. Up to 211 (86.6%) valuations did not have any justification for assumed discount rates.

The Panfil and Szablewski initiative should also be noted, according to which, at the same time valuations of the largest Polish companies from Warsaw Stock Exchange (WSE) were created [Panfil & Szablewski (ed.) 2014]. The authors' analysis of those reports found that:

1. Among the 19 examined recommendations, up to 14 times (74%) income valuation was used, and in every case (100%) FCFF was used.
2. In every case (100%) FCFF was discounted by the weighted average cost of capital.
3. In most cases (85.7 %) the variable discount rate was used.
4. In 2 valuations (14.3%) analysts declared the use of market weights, in 3 valuations (21.4%) book weights, while in 9 cases (64.3%) there were no references to the method of calculating the WACC weights. However, the authors' own analysis of the tables with financial forecasts of the balance sheets showed that in most cases book weights were used.

The authors' research clearly shows that analysts' prefer income valuation, above all. Both practitioners and theorists prefer FCFF among all available types of free cash flows. Statements in recommendations and valuations indicate that these flows are discounted with the correct discount rate – WACC. In most observed cases, for unknown reasons, there was a lack of information about how the projected capital structure of the valued companies was determined. Therefore, one open question still remains: whether the capital structure is estimated correctly, that is, in compliance with the DCF model's algorithm.

3. Research methodology

Based on this paper's evidence, there is no doubt that the truth of the equation (5) using the income valuation methods based on FCFE, FCFD and FCFF is dependent on the calculation method of the company's forecasted capital structure⁸. The method of WACC calculation adopted in this paper provides the truth of the equation (5), regardless of how the cost of each capital, which finances the valued company, will be assessed.

Prior to the calculation of the capital structure, the equation of DCF valuation should be paid particular attention to. The literature usually presents it in the following specific form:

$$V = \sum_{i=1}^n \frac{FCF_i}{(1+r_i)^i} + \frac{FCF_n \cdot (1+g)}{(1+r_n)} \quad (6)$$

where:

V – enterprise value,

FCF_i – Free Cash Flow in period i ,

r_i – discount rate in period i , appropriate for used type of FCF,

g – growth rate in residual period.

This formula applies only if $\frac{\sum_{i=1}^n r_i}{n} = r_i$, that is when the discount rate is constant throughout the period of detailed forecasts and beyond. If, on the other hand, the discount rate is variable, and this is the most common case when using WACC⁹, then the formula for the DCF valuation should be used in its general form:

$$V = \sum_{i=1}^n \frac{FCF_i}{\prod_{i=1}^n (1+r_i)} + \frac{FCF_n \cdot (1+g)}{\prod_{i=1}^n (1+r_i)} \quad (7)$$

⁸ There are also other ways of DCF valuation methods reconciliation [Gentry, Reilly & Sandretto 2003; Gentry & Reilly 2007]. Their approach involves estimating V_E by the FCFE, using a discount rate r_e and estimating V_D by the FCFD, using a discount rate $r_d \cdot (1 - t)$. Then they split V into two parts: the detailed forecast period (V_{DF}) and the residual period (V_{RV}), and they estimated the value of the company resulting from the period of a detailed forecast using the FCFF, with an estimated WACC. It was assumed that, on the basis of data which they collected, the equation $V_E + V_D = V_{DF} + V_{RV}$ makes it possible to calculate V_{RV} . Knowing V_{RV} as well as the last FCFF from detailed forecast period and having earlier used fixed WACC rate the growth rate can be calculated from the formula for the residual value. However, it should be noted that in such a defined reconciliation process, the growth rate of the company becomes the output data, not the input data, and it is computed only to ensure the compatibility of the equation (5).

⁹ Research indicates that in the Polish practice, 68% of valuations use a variable discount rate [Głębocki et al. 2011].

Formula (7) provides correct results even if the discount rate is variable. The assumption of the fixed discount rates during the detailed forecasts and beyond may, in practice, be too far-reaching and, in the case of the valuation of the sequential process, can cause incompatibility between the direct and sequential valuation.

The methods of Free Cash Flows calculation are presented based on the formulas provided by Damodaran [1994, 2012]:

$$FCFE = EBIT - I - (EBIT - I) * t + D\&A - CAPEX - \Delta WC + \Delta ND \quad (8)$$

$$FCFD = I * (1 - t) - \Delta ND \quad (9)$$

$$FCFF = EBIT - EBIT * t + D\&A - CAPEX - \Delta WC \quad (10)$$

where:

EBIT – Earnings Before Interests and Tax,

I – Interests,

t – tax rate,

D&A – Depreciation and Amortization,

CAPEX – Capital Expenditures,

ΔWC – Working Capital change,

ΔND – Net Debt change.

It is worth noting that $FCFF = FCFE + FCFD$.

In the empirical evidence the following was adopted:

1. The book value of equity at the valuation date, in the amount of 8 million currency units.
2. The book value of debt at the valuation date, in the amount of 2 million currency units.
3. The value of non-operating assets at the valuation date, in the amount of 0 currency units.
4. The variable cost of equity in each period of the detailed forecast, as shown in Table 2.
5. The variable cost of debt in each period of detailed forecast, as shown in Table 2.
6. The variable tax rate in each period of detailed forecast, as shown in Table 2.
7. The growth rate after the detailed forecast period at level of 4%.
8. The five-year detailed forecast period ($n = 5$).

For each period of detailed forecast operating profit, depreciation & amortization, investment in fixed capital, investment in working capital, and change in debt were estimated. Results of these projections are presented in Table 3.

The book value of the debt for each forecast period was estimated using the formula:

$$DBV_t = DBV_{t-1} * \Delta ND \quad (11)$$

where:

DBV_t – Book Value of Debt in period t ,

DBV_{t-1} – Book Value of Debt in period $t-1$.

The book value of equity for each forecast period was estimated using the formula:

$$EBV_t = EBV_{t-1} + NI_t - FCFE_t \quad (12)$$

where:

EBV_t – Book Value of Equity in period t ,

NI_t – Net Income in period t ,

$FCFE_t$ – FCFE in period t .

4. Research findings

Tables 2-8 present results for each period of detailed analysis, whereas Table 9 shows the final result of conducted study. Table 2 contains assumed cost of equity, cost of debt and tax rate. The results of the formulas (11) and (12) are shown in Table 5. Other cash flow components are calculated as indicated in Table 3. Calculated FCFE, FCFD and FCFF are shown in Table 4. Discount processes using FCFE, FCFD are presented in Tables 5 and 6, respectively. Table 7 presents the calculation of company's future capital structure, which is used in discount process of FCFF in Table 8. Formulas 13 and 14 show the consideration of non-operating assets in valuation algorithm. The input data are shown in bold.

Table 2. Estimated cost of equity, cost of debt and tax rate

	t1	t2	t3	t4	t5
re	0.0950	0.0950	0.0980	0.0980	0.1000
rd	0.0520	0.0500	0.0510	0.0610	0.0700
t	0.1900	0.2000	0.1900	0.1850	0.2000
rd*(1-t)	0.0421	0.0400	0.0413	0.0497	0.0560

Source: Authors' own derivation.

Table 3. Forecasts and calculations of components of cash flows

	t1	t2	t3	t4	t5
EBIT	650	720	880	1 000	1 010
$I = rd*DBV_{(t-1)}$	104	131	174	269	326
$GP = (EBIT - I)$	546	589	706	731	684
$EBIT*t$	124	144	167	185	202
$NOPAT = EBIT - EBIT*t$	527	576	713	815	808
$GP*t$	104	118	134	135	137
$NI = GP - GP*t$	442	471	572	596	547
D&A	210	210	228	229	235
CAPEX	410	430	400	411	460
ΔWC	100	101	106	113	109
ΔND	624	787	1 000	250	186
$TS = I*t$	20	26	33	50	65

Source: Authors' own derivation.

Table 4. Calculated free cash flows

	t1	t2	t3	t4	t5
FCFE	766	937	1 294	551	399
FCFD	-540	-682	-859	-31	75
FCFF	227	255	435	520	474

Source: Authors' own derivation.

Table 5. Equity valuation using FCFE method

	t1	t2	t3	t4	t5
FCFE	766	937	1 294	551	399
re	0.0950	0.0950	0.0980	0.0980	0.1000
RV					6 923
Disc.Ratio	0.9132	0.8340	0.7596	0.6918	0.6289
DFCFE (t0)	700	782	983	381	251
DRV (t0)					4 354
\sum DFCFE					7 450

Source: Authors' own derivation.

Table 6. Debt valuation using FCFD method

	t1	t2	t3	t4	t5
FCFD	-540	-682	-859	-31	75
$rd*(1-t)$	0.0421	0.0400	0.0413	0.0497	0.0560
RV					4 848
Disc.Ratio	0.9596	0.9227	0.8861	0.8441	0.7993
DFCFD (t0)	-518	-629	-761	-26	60
DRV (t0)					3 875
\sum DFCFD					2 000

Source: Authors' own derivation.

Table 7. Forecasted capital structure from DCF model

	t0	t1	t2	t3	t4	t5
EDCFV	7 450	7 392	7 157	6 564	6 657	6 923
DDCFV	2 000	2 624	3 411	4 411	4 661	4 848
EntDCFV	9 450	10 016	10 568	10 976	11 318	11 771
EDCFW	0.7884	0.7380	0.6772	0.5981	0.5882	0.5882
EDCFW	0.2116	0.2620	0.3228	0.4019	0.4118	0.4118

Source: Authors' own derivation.

Table 8. Enterprise valuation using FCFF method

	t1	t2	t3	t4	t5
FCFF	227	255	435	520	474
WACC	0.0838	0.0806	0.0797	0.0786	0.0819
RV					11 771
Disc.Ratio	0.9227	0.8539	0.7908	0.7332	0.6777
DFCFV (t0)	209	218	344	381	321
DRV (t0)					7 977
Σ DFCFF					9 450

Source: Authors' own derivation.

The value of equity is the sum of measured operating and non-operating assets financed by equity. According to the DCF valuation, it can be represented by the formula:

$$V_E = \sum_{i=1}^{\infty} DFCFE_i + NOA_0 \quad (13)$$

where:

$\sum_{i=1}^{\infty} DFCFE_i$ – sum of discounted FCFE, as shown in Table 6,

NOA_0 – non-operating assets at the valuation date¹⁰.

The value of the enterprise is the sum of measured operating and non-operating assets. According to the DCF valuation, it can be represented by the formula:

$$V = \sum_{i=1}^{\infty} DFCFF_i + NOA_0 \quad (14)$$

where:

$\sum_{i=1}^{\infty} DFCFF_i$ – sum of discounted FCFF, as shown in Table 9.

Table 9. Equity value in direct and sequential process of DCF valuation

	t0
V_E	7 450.387299843150000
$V - V_D$	7 450.387299843150000

Source: Authors' own derivation.

¹⁰ Non-operating assets are assumed to be financed only by equity.

As shown, the use of income weights ensures the consistency of equation $V_E + V_D = V$. In this approach, the capital structure is derived from the value of equity and debt, estimated using a DCF model. This means that the future capital structure of the company depends on the expected discounted cash flows, valued at a given moment in time, which economically justifies the adoption of such an approach for determining weights for company valuation. According to this method, weights for WACC in year t should be calculated as:

$$W_{e_t} = \frac{\sum_{i=t-1}^{\infty} DF CFE_i}{\sum_{i=t-1}^{\infty} DF CFE_i + \sum_{i=1-1}^{\infty} DF CFD_i} \quad (15)$$

$$W_{d_t} = \frac{\sum_{i=t-1}^{\infty} DF CFD_i}{\sum_{i=t-1}^{\infty} DF CFE_i + \sum_{i=t-1}^{\infty} DF CFD_i} \quad (16)$$

5. Conclusions

The literature review, analysis of valuation practices and the authors' own research indicate that what was once clearly defined because it resulted from the transfer of achievements of science to the social and economic science has since become subjective and ambiguous. The conclusion drawn from the conducted analysis is that the key to the reconciliation process, and thus DCF model coherency, is to apply the appropriate method of capital structure estimation during the calculation of weighted average cost of capital (WACC). This capital structure will henceforth be referred to as 'income weights'. The postulate of the use of income weights presented in this paper is also found by Fernandez [2015], who points out that what is commonly written as $E + D$ are neither book values, nor market values. He stated that the correct values to determine the future structure of the capital should be derived from the DCF model itself.

The determination of WACC using income weights proves that the algorithm of DCF valuation is consistent no matter:

1. Which income valuation process will be used: direct or sequential.
2. How the cost of equity in each forecast period will be determined.
3. How the cost of debt in each forecast period will be determined.
4. How the growth rate during the residual period will be determined.
5. How the debt change will be determined.

Company-related value variables, in particular the cost of debt, may or may not be correlated with the size of the company or the level of its debt. In relation to them, the method of WACC calculation using the income weights is independent, which makes it very useful in the practice of valuation.

The examination was conducted using MS Excel 2013 software. Interestingly, during the algorithm tests, when input data were amended many times, an inconsistency in obtained valuations results could be sporadically observed. The authors' own research and consultation with experts in mathematics and IT revealed that the described feature is associated not with an error in the presented algorithm. It is an Excel problem in the field of numerical arithmetic¹¹, which is also confirmed by the official statement of the Microsoft Corporation [www 3].

It should be noted that the results of this paper do not mean that a 'golden valuation formula' has been found, which always allows one to accurately and objectively 'calculate' the value of the company. This process continues to include highly subjective elements that are predictions. However, this by itself does not release analysts from the liability to use the analytical tool – the DCF model – correctly and precisely. Analysts valuating the company can still use the market or book weights, but they have to bear in mind that it is their responsibility to substantively justify why $5 - 2 = 3$, but $3 + 2$ does not necessarily equal 5.

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¹¹ The problem is the storage of only 15 significant digits, which is determined in accordance with the specifications of the Institute of Electrical and Electronics Engineers. More information about floating-point numbers and the IEEE 754 specification can be found on the official website [www 4].

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