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Further evidence on the validity of CAPM: The Warsaw Stock Exchange application

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Abstract

Aim/purpose – The purpose of the research is to verify the Capital Asset Pricing Model (CAPM) in the Polish capital market based on a conventional and downside risk approach.

Design/methodology/approach – The author in this study, using individual securities and portfolios, compares the unconditional risk-return relationships with the conditional risk, estimated in up and down market using realised returns in cross-sectional regressions. Except for a beta coefficient, the CAPM is tested with co-skewness as a higher order co-moment and downside betas as a risk measure in a downside approach.

Findings – The unconditional regressions give evidence of existing risk premium associated with co-skewness and downside beta, and confirmed the validity of the downside CAPM. The author, based on conditional relations, found that risk-return relations depend on the state of the stock market. The average premium for systematic risk in term of beta coefficient is significantly positive in up market periods and significantly negative in down market periods. The use of conditional models did not explicitly confirm the suitability of co-skewness in asset pricing.

Research implications/limitations – The main implications include the fact that the conventional beta coefficient is an appropriate risk measure when we consider using it separately for up and down market. A valuable extension of this research would be a benchmarking analysis to compare results for the Polish capital market against other emerging and developed markets.

Originality/value/contribution – The author in this paper proposes an alternative approach to testing risk-return relationships based on the CAPM in comparison to commonly used tests founded upon joint estimations of these relationships in periods of both positive and negative market excess return. The noteworthy contribution of this study is an application of the downside beta coefficient and the co-skewness coefficient in cross-sectional regressions.

Keywords: downside risk, co-skewness, conditional relationship, CAPM.

JEL Classification: G12, G32.

1. Introduction

Asset pricing is a set of activities for determining the value of a given asset for the currently specified moment or period. The prices are sufficiently approximated by their values only in sustainable and efficient markets (Fama, 1970). In the event of market imbalances, when the differences between effective demand and supply are significant, the market prices will be overpriced or underpriced. The capital asset pricing models are a method of determining the equilibrium prices of securities depending on the risk they represent. These models identify the sources and measures of risk appropriate from the point of view of the portfolio theory. Empirical confirmation of the relationship between profitability and their risk is important for decision-making processes regarding the selection of portfolios and for determining the forecasts of returns for selected assets. One of the most well-known and widely employed models of asset pricing in the financial theory is the Capital Asset Pricing Model (CAPM) proposed independently by Sharpe (1964), Lintner (1965) and Mossin (1966). The model assumes that the risk of any investment for which investors require some compensation is given by the contribution of the asset returns to non-diversifiable market risk, measured by the beta coefficient. Creators of the security market line define positive relationship between betas and returns.

The CAPM testing encounters many difficulties which are the result of the adopted statistical assumptions, including the normality of returns' distributions or treating variance as the main measure of risk (Markowitz, 1959). Nevertheless, the distributions are not often normal and utility function of investors' wealth is not quadratic (Piasecki & Tomasik, 2013). It implies that investors should consider more moments of these distributions, not only the mean and variance. Another assumption of CAPM is that investors behave equally towards upside and downside risk. However, investors perceive deviations from the threshold rate differently (Bawa & Lindenberg, 1977) and, therefore, they use

the utility function based on rather the lower partial moments (LPM) than variance. Researchers in many studies on CAPM rejected its validity, or even considered the theory empirically unverifiable (Cheung & Wong, 1992; Ostermark, 1991; Żarnowski & Rutkowska, 2012). In other words, researchers in the previous tests assumed that relationships between the systematic risk and the expected return are independent of market conditions and a variance is the only relevant risk measure that investors should consider in their investment decisions.

The above problems with CAPM assumptions highlight the need for different approaches to testing this model than in most tests as they attempt to estimate the market risk premium. In this paper, the author proposes the test of CAPM in three aspects constituting research problems and theses. First, the risk associated with achieving rates of return below the assumed level (downside risk) is priced on the Polish capital market. Second, the CAPM postulate in the context of the beta coefficient as an appropriate measure of risk is related to the condition of the stock market. At this point, the author assumes that the effect of beta coefficient is dependent on the positive or negative trend of the market. The insignificance of relationship in past studies between beta and returns is due to the fact that periods of positive realised returns can be offset by periods of negative realised returns. Third, investors are rewarded for bearing a risk associated with the co-moments, in particular co-skewness of returns distribution. These three statements are the main theoretical premises for empirical investigation in this work.

The management in a sustainable market using downside risk and higher moments of returns acquires special significance taking into account unexpected events related to the economy of a country, political events or financial crises. The Warsaw Stock Exchange (WSE) is the biggest exchange of financial instruments in Central and Eastern Europe and the one of the most recognisable Polish financial institutions. On the WSE, there are quoted over 3,000 instruments and over 1,000 domestic and international issuers. Even though the Polish capital market is relatively young (28 years) on 29 September 2017 FTSE Russell has announced the upgrade of Poland from Emerging Market to Developed Market status. This decision should stabilise the Polish stock market. Poland has significantly improved the infrastructure and quality of the capital market, which is associated with the economic development of the country. The changes concern, including regulators' monitoring, rights of shareholders, transfers of capital and dividends, and the liquidity of the stock market. The Polish capital market may be an appropriate illustration to study of asset risk in the context of above measures due to these changes. Many researchers provided that parameters of

the return distributions are different in emerging and developed markets (Bekaert et al., 1998) and the results in relation to risk premiums for downside measure in these markets are inconsistent (Estrada, 2002; Rashid & Hamid, 2015). Moreover, the Polish capital market is not characterised by high liquidity like developed capital markets, such as the USA or the UK, which may cause volatility of market trends. However, research shows that liquidity and volatility causality is bidirectional. Though, it should be acknowledged that liquidity is more often Granger causes volatility than volatility causes liquidity (Będowska-Sójka & Kliber, 2019).

The main objective of this study is to test the standard and extended CAPM relations between systematic risk measures and realised returns for single companies quoted on the Polish capital market and equally-weighted portfolios. The study proposes the analysis of unconditional and conditional relationships, considering positive and negative market excess return. The unconditional relations will be estimated in the downside approach as well.

This study is organised as follows: section 2 reviews the literature on downside risk, higher moments and conditional versus unconditional risk-return relationships. Section 3 describes the risk measures and methodology with hypotheses of conditional and unconditional cross-sectional regression. Research findings and discussion is in section 4, and section 5 offers conclusion.

2. Literature review

One of the concepts of model specification and thus its verification is an approach in which relationships between the systematic risk and the expected return depend on market conditions. The first study using the above approach was an empirical evidence (Pettengill, Sundaram, & Mathur, 1995) where the relationship between rates of return and beta coefficients with high and low levels of this coefficient are conditioned to the relationship between the realised market rate of return and the risk-free rate. This approach is in contrast to the unconditional procedure proposed by Fama & MacBeth (1973). Pettengill et al. (1995) demonstrated that when excess returns are positive, the CAPM usually predicts positive relation between beta and returns. However, when excess realised returns are negative, the CAPM predicts an inverse relation between beta and return. They estimated the beta premium separately in the case of positive and negative market excess return, and tested the hypotheses that the risk premiums are statistically significantly different from zero and, respectively, positive in up markets and negative in down markets. This statement is relevant in the

context of testing systematic relations between returns and beta coefficients, especially when the market excess return was negative. That approach to this phenomenon flows from negative attitude of investors towards downside risk and argues the case for greater attention to the need for more CAPM research. The demonstration of a statistical significance of market risk premium in conditional CAPM relationships allows to treat the beta as an important and useful measure of risk. This approach was reflected in many studies. Research showed positive risk-return relationships which is consistent with the postulates of the CAPM (Bilgin & Basti, 2014; Fletcher, 2000; Jagannathan & Wang, 1996; The-riou, Aggelidis, Maditinos, & Šević, 2010; Trzpiot & Krężolek, 2006). The results obtained by Galagedera, Henry, & Silvapulle (2003) based on conditional regression illuminate often pessimistic conclusions regarding the equilibrium model. They confirmed that the risk-return relationship is conditioned by the sign of the market excess return.

Another issue of CAPM testing in the context of a bull and bear market was the stability of CAPM parameters. Fabozzi & Francis (1977) supported the single beta CAPM. They found no significance in using two betas, one for the bull market and other for the bear market. Not only can the conditional cross-sectional regression be considered in the context of market condition, but also market volatility as changes in market movement. Galagedera & Faff (2005) demonstrated the impact of market volatility on the risk premiums and found that the market risk premium for beta in the three market volatility regimes was statistically significant.

Moreover, the risk is often perceived by investors only in the form of negative deviations of returns from the assumed rate of return. Risk-averse investors differently perceive deviations below and above, for example, the expected value. In this case, semi-variance is a better measure of risk than the measures based on variance because it treats the risk as a real loss as opposed to gain of upside risk. Such a perspective allows for developing the concept of downside risk which main measures are the downside beta coefficients based on LPM. This approach takes on special significance in the case of abnormal returns distributions and their asymmetry.

Downside betas are the systematic measures estimated over the periods for which the market return is below risk-free rate (Bawa & Lindenberg, 1977), mean (Hogan & Warren, 1974) or zero as the threshold. Investors with aversion to downside risk will require a positive significant premium for bearing this type of risk. Many studies on developed and emerging markets showed that downside

measures better explain variability in the cross-section of returns than conventional ones (Estrada, 2002, 2007; Post & van Vliet, 2006). The studies using individual securities also demonstrate that downside risk measures better explain securities returns than the beta coefficient (Alles & Murray, 2013; Pedersen & Hwang, 2007). Ang, Chen, & Xing (2006), based on companies listed on the NYSE, AMEX and NASDAQ, showed that investors are rewarded with a market premium for the downside risk, which means that the assets with higher downside beta coefficients reach higher rates of return on average. Xu & Pettit (2014) using similar to Ang et al. (2006) methods of estimation of upside and downside beta showed that returns are strongly correlated with downside betas and weakly correlated with upside betas.

The occurrence of skewed returns distributions prompts to use non-quadratic utility function for an investor extended by higher moments (Scot & Horvath, 1980). Furthermore, the quadratic utility function implies an increasing risk aversion, whereas it is more appropriate to assume that risk aversion decreases when the wealth increases. In asset pricing, other than mean and variance systematic risk measures called moments or in particular co-moments, such as co-skewness, must be considered. In the pricing theory, as a form of verification the three and four-moment CAPM are widely used based on higher-order co-moments (co-skewness and co-kurtosis) (Kraus & Litzenberger, 1976). Barone-Adesi (1985) showed a quadratic model to test the three-moment CAPM. Harvey & Siddique (1999) proposed an analysis of the effect of co-skewness on asset prices. Many studies confirm that higher co-moments are risk factors influencing asset returns and better predicting returns than the mean-variance approach, both in developed and emerging markets (Fernandes, Fonseca, & Iquiapaza, 2018; Galagedera et al., 2003; Mora-Valencia, Perote, & Arias, 2017; Neslihanoglu, Sogiakas, Mccoll, & Lee, 2017; Teplova & Shutova, 2011). Chiang (2016) investigated skewness and co-skewness pricing for bond return and suggested these measures are at least conditionally significant. De Roon & Karehnke (2016) draw attention to the possibility of use and interpretation of co-skewness in portfolio choice and connecting this theory with the asset pricing.

The author of this research adopted a conditional approach by Pettengill et al. (1995) to verify the non-standard CAPM version with higher order co-moment. In the Polish literature, there are few studies devoted to pricing of downside risk and co-moments especially in conditional relations of CAPM. In this paper, the standard and extended version of CAPM using co-skewness in the conventional framework is proposed.

3. Research methods and procedure

3.1. Unconditional relationships under conventional framework

The study of relations between beta coefficients and realised rates of return was carried out in a two-step procedure (Fama & MacBeth, 1973). In the first stage, based on all observations of the sample, the beta coefficients of the securities were estimated using the Sharpe's single-index model given as:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \xi_{it} \quad (i = 1, \dots, N; t = 1, \dots, T), \quad (1)$$

where:

R_{it}, R_{Mt} – rates of return for the i -th asset and the rate of return of the market portfolio, respectively,

α_i – constant term,

β_i – beta coefficient of i -th asset,

ξ_{it} – random error term of i -th equation.

In the second stage, the regression analysis was based on cross-sectional rows where the dependent variables were realised excess returns on assets and the independent variables were the beta coefficients of assets estimated in the first stage of the procedure. The unconditional cross-sectional relationships were estimated for each period of the sample as follows (Nurjannah, Galagedera, & Brooks, 2012):

$$R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t} \hat{\beta}_i + \eta_{it} \quad (i = 1, \dots, N; t = 1, \dots, T), \quad (2)$$

where:

R_{ft} – risk-free rate,

$\lambda_{0t}, \lambda_{1t}$ – parameters of t -th equation,

η_{it} – random error term of t -th equation.

From the relationships (2) some testable implication of the CAPM can be formulated. The average risk premium λ_1 associated with the market risk premium (beta coefficient) for whole study period should take positive values. Hypotheses regarding this parameter (Tang & Shum, 2003) are:

$$\begin{aligned} H_0: E(\lambda_1) &= 0 \\ H_1: E(\lambda_1) &> 0. \end{aligned} \quad (3)$$

Finally, because the asset uncorrelated with the market portfolio has the expected rate of return equal to the risk-free rate, constant term of the relationship (2) should not be significantly different from zero which means that:

$$\begin{aligned} H_0: E(\lambda_0) &= 0 \\ H_1: E(\lambda_0) &\neq 0. \end{aligned} \quad (4)$$

The above hypotheses were tested using one mean significance t test with a one-sided or two-sided critical area.

3.2. Three-moment unconditional pricing models

The portfolio analysis restricts investors' preferences to the first two moments, which characterise the distribution of rates of return. However, intuitively, investors will prefer positive returns' distributions giving them a considerable chance of achieving high rates of return. Investors in the process of creating a portfolio acknowledge the marginal contribution of a given asset to the portfolio asymmetry. Therefore, they prefer a security that increases the right-hand asymmetry of portfolio return rates, rather than a security extending left tail of distribution. Under the equilibrium conditions, investors are willing to pay for shares inclusion of which in the portfolio increases its right-hand asymmetry. Therefore, investors require a risk premium considering securities with a 'negative' contribution to the asymmetry of portfolio returns. The measure that examines the contribution of a given asset to the asymmetry of the market portfolio is co-moment called co-skewness γ_i which is formulated as follows (Cheng, 2005):

$$\gamma_i = \frac{E[(R_{it} - E(R_i))(R_{Mt} - E(R_M))^2]}{E[(R_{Mt} - E(R_M))]^3}. \quad (5)$$

The assets with positive co-skewness will make the market portfolio distribution more skew in the direction of the asymmetry of the market portfolio. The sign of the premium for the risks described in co-skewness will depend on the asymmetry of market portfolio and is expected to be opposite to the sign of skewness of market portfolio distribution. In a situation where the skewness of market return distributions is negative investors will require a positive premium for co-skewness, while if the market portfolio distribution is right-hand skewed, investors will be willing to pay for the contribution of assets with a positive co-skewness to the portfolio. Then, a negative risk premium should be expected.

The unconditional cross-sectional relationships using co-skewness were estimated for each month of the sample as follows:

$$R_{it} - R_{ft} = \lambda_{0t} + \lambda_{2t}\hat{\gamma}_i + \eta_{it} \quad (i = 1, \dots, N; t = 1, \dots, T), \quad (6)$$

$$R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \lambda_{2t}\hat{\gamma}_i + \eta_{it} \quad (i = 1, \dots, N; t = 1, \dots, T), \quad (7)$$

where:

$\hat{\gamma}_i$ – estimate of the co-skewness in the distribution of i -th asset returns.

Sets of hypotheses regarding parameters λ_2 are as follows:

$$\begin{aligned} H_0: E(\lambda_2) &= 0 \\ H_1: E(\lambda_2) &> 0 \text{ when } As_M < 0 \end{aligned} \quad (8)$$

and

$$\begin{aligned} H_0: E(\lambda_2) &= 0 \\ H_1: E(\lambda_2) &< 0 \text{ when } As_M > 0, \end{aligned} \quad (9)$$

where:

As_M denotes the skewness of market portfolio.

These hypotheses were tested using one mean t test with a one-sided critical area.

3.3. Unconditional relationships under downside framework

In this section, the role of the downside systematic risk in asset pricing is considered. The conception of systematic risk measures in the context of the downside risk in this part of the work will be based on the second lower partial moment. One of the first measures of this type that was proposed in the literature is the downside beta coefficient defined by Hogan & Warren (1974) and Bawa & Lindenberg (1977) and it is expressed as follows:

$$\beta_i^{HW} = \beta_i^{BL} = \frac{E[(R_{it} - R_f) \min(R_{Mt} - R_f; 0)]}{E[\min(R_{Mt} - R_f; 0)]^2} \quad (10)$$

where:

R_{it}, R_{Mt}, R_f – the return in time t for security i , the market portfolio return in time t and the risk-free rate, respectively.

In the downside framework, the key factor of interpretation and in assessing downside risk is a threshold rate. In the theory, there are many downside beta coefficients distinguished with different formulas (Estrada, 2002) and threshold

rates. In contrast to downside beta in relation (10) investors may treat risk as downside deviations below the threshold that is the average market portfolio returns. This approach was proposed by Harlow & Rao (1989), formulating the downside beta coefficient as follows:

$$\beta_i^{HR} = \frac{E[(R_{it} - E(R_i)) \min(R_{Mt} - E(R_M); 0)]}{E[\min(R_{Mt} - E(R_M); 0)]^2} \quad (11)$$

In this study the estimation of unconditional relations is based on the downside beta that is defined by relation (11). Then the unconditional cross-sectional D-CAPM (downside CAPM) relationships were estimated for each month of the sample as follows (Nurjannah et al., 2012):

$$R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t} \hat{\beta}_i^{HR} + \eta_{it} \quad (i = 1, \dots, N; t = 1, \dots, T). \quad (12)$$

The testable hypothesis about parameter λ_1 is similar to the one defined for unconditional model described in relation (2).

3.4. Conditional relationships under conventional framework

A conditional approach to testing the CAPM was proposed by Pettengill et al. (1995). Conditional, due to the sign of the market excess return, the CAPM equation in the testable version is in the form:

$$R_{it} - R_{ft} = \delta \lambda_{0t}^U + (1 - \delta) \lambda_{0t}^D + \delta \lambda_{1t}^U \hat{\beta}_i + (1 - \delta) \lambda_{1t}^D \hat{\beta}_i + \eta_{it}, \quad (13)$$

where:

δ – a dichotomous variable used to indicate the positive and negative market excess return, then $\delta = 1$ if $(R_{Mt} - R_{ft}) > 0$ and $\delta = 0$ if $(R_{Mt} - R_{ft}) < 0$,

$\lambda_{0t}^U, \lambda_{0t}^D, \lambda_{1t}^U, \lambda_{1t}^D$ – parameters of t -th equation,

η_{it} – random error term of t -th equation.

The average estimate of λ_1^U should be statistically significantly greater than zero in the periods with positive excess market return and the average estimation of λ_1^D should be statistically significantly less than zero in the periods with negative excess market return. The set of hypotheses is as follows (Pettengill et al., 1995):

$$\begin{aligned} H_0: E(\lambda_1^U) &= 0 \\ H_1: E(\lambda_1^U) &> 0 \end{aligned} \quad (14)$$

and

$$\begin{aligned} H_0: E(\lambda_1^D) &= 0 \\ H_1: E(\lambda_1^D) &< 0 \end{aligned} \quad (15)$$

Rejection of null hypotheses in both cases will indicate the occurrence of systematic, conditional relations between the beta coefficients and the realised assets returns. Rejection of the null hypothesis in one case only is not enough to prove conditional risk-return relationships.

Similarly, to the unconditional approach, the parameters of extended CAPM versions were estimated for each month based on the cross-sectional regressions of the form (Galagedera et al., 2003):

$$\begin{aligned} R_{it} - R_{ft} &= \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{2t}^U\hat{\gamma}_i + (1 - \delta)\lambda_{2t}^D\hat{\gamma}_i + \eta_{it} \quad (16) \\ R_{it} - R_{ft} &= \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\hat{\beta}_i + (1 - \delta)\lambda_{1t}^D\hat{\beta}_i + \delta\lambda_{2t}^U\hat{\gamma}_i + \\ &+ (1 - \delta)\lambda_{2t}^D\hat{\gamma}_i + \eta_{it}, \quad (17) \end{aligned}$$

where:

$\lambda_{0t}^U, \lambda_{0t}^D, \lambda_{1t}^U, \lambda_{1t}^D, \lambda_{2t}^U, \lambda_{2t}^D$ – parameters of t -th equation,

η_{it} – random error term of t -th equation.

Expected signs of estimated parameters in the periods of positive and negative market excess return present the following sets of hypotheses:

$$\begin{aligned} H_0: E(\lambda_2^U) &= 0 & H_0: E(\lambda_2^D) &= 0 \\ H_1: E(\lambda_2^U) &> 0 & \text{and } H_1: E(\lambda_2^D) &< 0 \end{aligned} \quad (18)$$

when in periods of $(R_{Mt} - R_{ft}) > 0$ is $As_M < 0$

and

$$\begin{aligned} H_0: E(\lambda_2^U) &= 0 & H_0: E(\lambda_2^D) &= 0 \\ H_1: E(\lambda_2^U) &< 0 & \text{and } H_1: E(\lambda_2^D) &> 0 \end{aligned} \quad (19)$$

when in periods of $(R_{Mt} - R_{ft}) > 0$ is $As_M > 0$.

The case of rejecting null hypotheses both in (18) and (19) means that conditional relationships between rates of return and systematic co-skewness are legitimate.

4. Research findings and discussion

A dataset for empirical analyses of the CAPM relationships were a time series of monthly simply returns of individual securities, which belong to all macrosectors, quoted on the Warsaw Stock Exchange. The sample period is from

January 2010 to December 2017 and represents 96 observations. The full-time series in the analysed period were characterised by 207 companies. The WIG index is used as the market portfolio approximation and the proxy for the risk-free rate was average-weighted money bills rate issued by the Polish National Bank. This rate equals to the reference rate. The tested sample period was characterised by symmetry as to the number of positive $(R_{Mt} - R_{ft}) > 0$ and negative $(R_{Mt} - R_{ft}) < 0$ market excess returns, 48 observations of each type. Estimations of the proposed relationships were carried out using equally-weighted portfolios as well. In that procedure 21 portfolios were formed by ascending sorted securities on the relevant measure of risk. These portfolios were comprised of 10 securities except for the last one which comprised 7 stocks with the highest values of risk measure.

The study of conditional and unconditional relations was preceded by the correlation analysis between average rates of return and the risk measures respectively. The results of the study are presented in Table 1.

Table 1. Correlation coefficients between measures of risk in whole sample

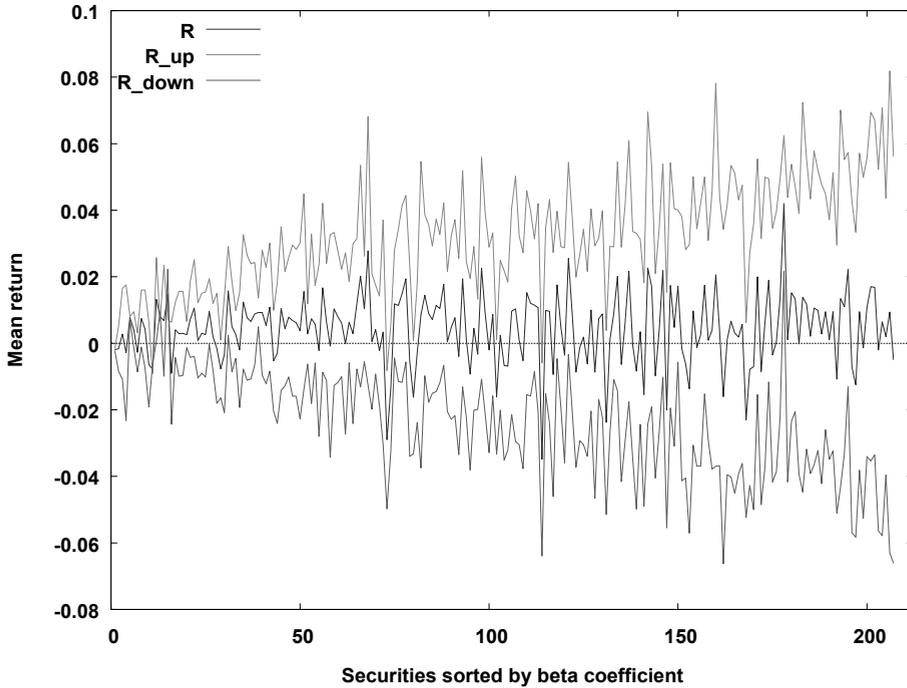
Measure	\bar{R}_i	β_i	β_i^{HR}	γ_i	Measure	\bar{R}_P
\bar{R}_i	1.000				\bar{R}_P	1.000
β_i	0.038	1.000			β_P	0.160
β_i^{HR}	0.122***	0.906*	1.000		β_P^{HR}	0.475**
γ_i	-0.193*	0.188*	-0.223*	1.000	γ_P	-0.441**

Notes: \bar{R}_i , \bar{R}_P are the average return of individual securities and portfolios, respectively; β_i , β_P , β_i^{HR} , β_P^{HR} are CAPM beta and downside beta for individual securities and portfolios, respectively; γ_i , γ_P are co-skewness for individual securities and portfolios, respectively; *, **, *** indicates significance at the 1%, 5% and 10%, respectively.

There is not any significant correlation between the CAPM beta and average returns for both individual stocks and portfolios. However, the significant positive correlation between the downside beta and average returns was found, which means the validity of downside CAPM (D-CAPM). The above correlation is statistically significant at the level of 0.1 for individual securities and at the level of 0.05 for portfolios. Investors assessing their assets based on CAPM should consider the sensitivity of these assets to downside risk.

In the context of conditional relationships, there can be observed a distinct positive relation (in periods of positive market excess return, up market) and negative relation (in periods of negative market excess return, down market) between beta coefficients and expected rates of return (Figure1).

Figure 1. Conditional and unconditional relations between expected returns and beta coefficients



As shown in Figure 1, the higher the beta values are, the higher the absolute average returns are. The author argues that the standard CAPM equation holds for the Polish capital market, but investors should consider it separately in up and down market rather than for aggregated data.

As far as co-skewness is concerned, it is an important aspect of risk both in the context of the impact of asymmetry on the rates of return and as another aspect of downside risk. The correlation between co-skewness and average returns is statistically significant and negative, which is consistent with the assumptions. In the sample period, the positive asymmetry of the market portfolio exists ($As_M = 0.08$), and therefore a negative risk premium is desirable. The obtained results are consistent with previous findings from study of Teplova & Shutova (2011) and Duc & Nguyen (2018).

In the further part of the research, unconditional cross-sectional regressions were estimated. The results of these estimations are presented in Table 2 and these ones do not allow for the rejection of the null hypothesis with a positive and statistically significant premium for market risk which is not consistent with

the CAPM postulates. Extended versions of this model also confirm that there is no positive significant market risk premium. The risk premium associated with co-skewness turned out to be negative and statistically significant at the significance level of 0.1 both for individual securities and portfolios. The sign of these premiums is opposite to the sign of the market skewness which is consistent with the theory. On the Polish capital market, in the sample period, the distribution of market portfolio is right-hand skewed, thus investors will be willing to pay for the contribution of assets with a positive co-skewness to the portfolio. Moreover, it is very interesting and surprising that the risk premium for co-skewness is significant in the absence of such significance in the event of premium for market risk. The unconditional results of significance of co-skewness contradict the previous research by Galagedera et al. (2003) and Lee, Robinson, & Reed (2008) where unconditional models with co-skewness did not explain realised returns.

Table 2. Estimates of unconditional CAPM relations

Coefficient	Individual securities			Equally-weighted portfolios		
	Mean	<i>t</i> -Stat	<i>p</i> -Value	Mean	<i>t</i> -Stat	<i>p</i> -Value
Model: $R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \eta_{it}$						
λ_{0t}	0.0035	1.453	0.149	0.0032	1,298	0.197
λ_{1t}	0.0011	0.200	0.421	0.0013	0.258	0.389
Model: $R_{it} - R_{ft} = \lambda_{0t} + \lambda_{2t}\hat{\gamma}_i + \eta_{it}$						
λ_{0t}	0.0043	0.940	0.349	0.0042	0.942	0.348
λ_{2t}	-0.0005	-1.335	0.092***	-0.0005	-1.299	0.098***
Model: $R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i + \lambda_{2t}\hat{\gamma}_i + \eta_{it}$						
λ_{0t}	0.0024	1.084	0.281	-0.0001	-0,018	0.985
λ_{1t}	0.0022	0.405	0.343	0.0867	0.590	0.278
λ_{2t}	-0.0005	-1.360	0.088***	-0.0005	-1.316	0.095***

Notes: *, **, *** indicates significance at the 1%, 5% and 10%, respectively.

In the next step of the analysis unconditional cross-sectional regressions were estimated under the downside framework for individual securities and portfolios sorted by the downside beta. The results are given in Table 3. The author found that the results obtained for the portfolios indicate a positive and signifi-

cant relationship between the downside risk and the realised rates of return. This relationships cannot be confirmed in the case of individual companies. Nevertheless, the author gives strong evidence of the risk premium associated with downside beta (results in Table 3 and Table 1). The downside approach has an advantage over conventional one. That means that assets more sensitive to downside market risk achieve relatively larger losses when market is decreasing. For this reason, these assets are not attractive for investors and they will require higher compensation to hold such assets.

Table 3. Estimates of unconditional D-CAPM relations

Coefficient	Individual securities			Equally-weighted portfolios		
	Mean	<i>t</i> -Stat	<i>p</i> -Value	Mean	<i>t</i> -Stat	<i>p</i> -Value
Model: $R_{it} - R_{ft} = \lambda_{0t} + \lambda_{1t}\hat{\beta}_i^{HR} + \eta_{it}$						
λ_{0t}	0.0014	0.638	0.525	-0.0001	-0.034	0.972
λ_{1t}	0.0033	0.677	0.250	0.0083	1.678	0.048**

Notes: *, **, *** indicates significance at the 1%, 5% and 10%, respectively.

The author in the last part of the research concentrated on the conditional cross-sectional regressions in which periods of positive and negative market excess return were not aggregated. The estimations of conditional relationships verified the hypotheses set out in section 3.4. The results of these estimations are presented in Table 4. The author uncovered that systematic effect of beta risk premium is the consequence of a separate treatment of periods with a negative and positive market excess return. In all models where the CAPM beta is an independent variable there is positive and significant risk premium in up market and negative and significant premium in down market, as it was expected.

The author found that the average estimation of the parameter λ_1^U in conditional relations was positive and in the range from 0.0327 to 0.0348 for securities and from 0.0292 to 0.0370 for portfolios and it was statistically significant at the level of significance $\alpha = 0.01$ and $\alpha = 0.05$. The average estimation of the parameter λ_1^D was negative and in the range from -0.0305 to -0.0260 for securities and from -0.0323 to -0.0267 for portfolios and it was statistically significant at the level of significance $\alpha = 0.01$ and $\alpha = 0.05$ as well. In conclusion, securities with high beta coefficients in periods with a positive market excess return (with a negative market excess return), achieve higher rates of return (lower rates of return) than securities with relatively lower beta coefficients.

Table 4. Estimates of conditional CAPM relations

Individual securities					Equally-weighted portfolios		
Market condition	Coefficient	Mean	t-Stat	p-value	Mean	t-Stat	p-value
Model: $R_{it} - R_{ft} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\hat{\beta}_i + (1 - \delta)\lambda_{1t}^D\hat{\beta}_i + \eta_{it}$							
Up market $\delta = 1$	λ_{0t}^U	0.0034	1.232	0.224	0.0073	0.884	0.381
	λ_{1t}^U	0.0327	6.180	0.000*	0.0292	2.190	0.017**
Down market $\delta = 0$	λ_{0t}^D	-0.0006	-0.179	0.231	0.0034	0.647	0.520
	λ_{1t}^D	-0.0260	-4.403	0.000*	-0.0323	-4.824	0.000*
Model: $R_{it} - R_{ft} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{2t}^U\hat{\gamma}_i + (1 - \delta)\lambda_{2t}^D\hat{\gamma}_i + \eta_{it}$							
Up market $\delta = 1$	λ_{0t}^U	0.0325	6.503	0.000*	0.0327	6.479	0.000*
	λ_{2t}^U	-0.0001	-0.069	0.472	-0.0001	-0.059	0.477
Down market $\delta = 0$	λ_{0t}^D	-0.0239	-4.626	0.000*	-0.0240	-4.605	0.000*
	λ_{2t}^D	-0.0009	-2.109	0.979	-0.0010	-2.156	0.981
Model: $R_{it} - R_{ft} = \delta\lambda_{0t}^U + (1 - \delta)\lambda_{0t}^D + \delta\lambda_{1t}^U\hat{\beta}_i + (1 - \delta)\lambda_{1t}^D\hat{\beta}_i + \delta\lambda_{2t}^U\hat{\gamma}_i + (1 - \delta)\lambda_{2t}^D\hat{\gamma}_i + \eta_{it}$							
Up market $\delta = 1$	λ_{0t}^U	0.0030	1.050	0.300	0.0013	0.164	0.870
	λ_{1t}^U	0.0348	5.371	0.000*	0.0370	2.980	0.002*
	λ_{2t}^U	-0.0006	-1.048	0.150	-0.0006	-0.950	0.173
Down market $\delta = 0$	λ_{0t}^D	0.0018	0.520	0.605	-0.0015	-0.198	0.844
	λ_{1t}^D	-0.0305	-5.361	0.000*	-0.0267	-2.361	0.011**
	λ_{2t}^D	-0.0004	-0.834	0.795	-0.0005	-0.891	0.803

Notes: *, **, *** indicates significance at the 1%, 5% and 10%, respectively.

The estimates of the risk premium related to co-skewness only partly correspond to the hypotheses and are not consistent with author's expectations. Distribution of market returns for periods with positive market excess return had positive asymmetry (0.364) and for periods with negative market excess return had negative asymmetry (-1.548). The average values of the co-skewness risk premium in three-moment conditional model were negative but statistically insignificant in up market. The average premiums for co-skewness in down market both securities and portfolios were negative, which is not consistent with the hypotheses regarding these parameters. These results are opposite to the findings in Galagedera et al. (2003). As far as the co-skewness coefficient is concerned, the unconditional three-model failed and conditional three-model held. This

means that the risk of co-skewness is independent of the market condition manifesting as positive and negative market excess return on the Polish capital market.

5. Conclusions

The capital assets pricing is still one of the mainstream studies in both emerging and developed capital markets. The author presents the proposal of an alternative study of the risk-return relationships in the context of CAPM.

5.1. Research contribution

This research provided some important findings. Considering the individual securities quoted on the Warsaw Stock Exchange and equally-weighted portfolios, there is the empirical evidence that beta coefficient fails to explain changes in stock returns according to the conventional approach of unconditional models. These results are consistent with the tests described in many previous papers, especially on emerging capital markets. For example, Galagedera et al. (2003), Nurjannah et al. (2012) and Thuy & Kim (2018) proved insignificant relationship between average or realised returns and CAPM betas.

The downside approach based on portfolios outperforms the conventional counterparts. The acceptable level of downside risk was found to be significant at 5% positive market premium. Downside beta turned out to be a more appropriate measure of systematic risk than the conventional beta coefficient in explaining the cross-section of portfolio returns. This conclusion was supported by, among others, Estrada (2007), Ang et al. (2006) and Chhapra & Kashif (2019). They confirmed that downside beta is a better predictor of portfolio returns than conventional beta. It may change the perception of risk by investors only in the variance context and thus investors require compensation for bearing downside measure.

The essential part of the research was an estimation of conditional relations between the realised rates of return and beta coefficients, separately for periods with a positive and negative market excess return. The obtained results of the tested hypotheses are confirmed by earlier studies in the literature and allow the author to formulate the following conclusions. Relations between the beta coefficients and realised rates of return are conditioned by the sign of the market excess return. The average value of systematic risk premium is significantly

higher than zero in periods of positive market excess return and significantly lower than zero in periods of negative market excess return. Similar results were confirmed on the Russian Stock Exchange (Teplova & Shutova, 2011) or Australian capital market Nurjannah et al. (2012). Nevertheless, earlier results for the Polish market are different from those presented in this paper and reject the CAPM model in both an unconditional and conditional model (Trzpiot & Kręzolek, 2006). The reason for this situation may be the fact that the capital market in Poland at that time was an emerging market with a little over 10 years of history. This resulted in a weak information efficiency of the assets listed on the Warsaw Stock Exchange.

The paper showed a weak but statistically significant risk premium for the co-skewness in unconditional cross-sectional regressions. For the whole sample period the asymmetry of market portfolio was positive thus negative sign of risk premium was expected. The use of conditional models did not explicitly confirm the suitability of co-skewness in asset pricing. In the up-market periods with positively skewed market portfolio distribution the risk premium for co-skewness was negative which was expected but not significant. In the down market, with negative asymmetry of market portfolio distribution, the risk premium for co-skewness was also negative which is not desirable. Such results are inverse to those obtained by Galagedera et al. (2003). Many markets have been found to have different results regarding co-skewness. This may lead to further research on various other conditions adopted in conditional models. One of such conditions from which inconsistencies in the results on emerging and developed markets may arise are the failure to accommodate market movements, especially for low and high regimes of market volatility.

5.2. Research implication

The risk-return relations on the Polish capital market are similar to those for emerging and even developed markets. The research provides academic as well as practical implications in the field of financial investment management. The obtained results in a given capital market prove that tests of CAPM using relationships which not consider market conditions, often lead to misleading conclusions about the validity of this model. The asset pricing based on the conventional beta coefficient is appropriate when we consider the use of this measure separately for up and down market. In addition, using realised returns as a proxy of expected returns as the cost of equity capital, the CAPM significantly captures

the market risk premium. Moreover, the significant pricing of the higher moment, which is co-skewness, puts the theory of investor's utility and the existence of a risk-averse attitude in a different light.

The received findings can be important to managers and institutions since it may allow them to extend the scope of measures used in making decisions in portfolio selection process and capital budgeting process. For instance, skewness can significantly change the optimal structure of the portfolio because the greater the strength of preferences regarding skewness is, the smaller the degree of portfolio diversification is. Furthermore, downside risk should deserve special attention of investors who estimate the cost of equities on the Polish stock market.

5.3. Research limitation and future works

The presented research does not exhaust the subject both in the context of the proposed measures of risk and the methodology of testing the CAPM model. Research limitations are related to the number of assets taken for the analysis (the compromise between the length of time series and the sample size). The presented proposal certainly requires an analysis of the robustness of the obtained results to changes in the study period or using panel data analysis. A valuable extension of research would be benchmarking analysis to compare results on the Polish capital markets against other emerging and developed markets.

Author's further research on the capital assets pricing will be expanded to other higher co-moments like co-kurtosis in both conventional and downside approaches and in different sub-samples.

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