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DETERMINISTIC CHAOS IN ECONOMIC PROCESSES MODELING

Summary: Modeling of economic processes is the subject of numerous studies and analyzes. There are a variety of methods and research tools used. Attempts to describe the functioning of economic processes are taken within multiple disciplines, e.g. economics, mathematics, psychology. A variety of theories and methods used are reflected in the diversity of the obtained results and forecasts. In order to predict the future behavior of the stock market or currency market, various models are designed, which never give full assurance of success and are usually burdened, with investment risk. One of the newer concepts of modeling economic processes, such as the stock market or the currency market, is the deterministic chaos theory. It is an attempt to move away from the idea of the efficiency of capital markets and currency markets, towards a more universal view of the mechanisms governing them. Characteristics features, imbalances and positive feedback mechanism in time, are reflected in the description with the use of non-linear dynamic systems. The publication discusses the selected issues using deterministic chaos theory to describe the operation of some economic processes. The paper includes issues related to the essence of deterministic chaos, selected examples of chaos theory to describe economic processes with particular emphasis on the capital market and currency market.

Keywords: deterministic chaos, capital markets, currency markets, chaos theory.

Introduction

As a result of research conducted over a period, it was thought that economic systems are simple, continuous, and linear. Consequently, most research was focused primarily on linear models, operating on the basis of a methodology consistent with the assumptions of the theory of equilibrium and rational expectations of economic entities. This approach has changed as a result of present global crises, which highlighted deficiencies in existing research theories, which are not reflected in the reality. As a result, the research practice has begun to focus on non-linear models, which also have started to be regarded as an effective form of depicting modern economic processes [Jakimowicz, 2013, p. 364].

One of the first researchers in the field of research on deterministic models (to describe cycles) were M. Kalecki, J. Tinberger and N. Kalder [Drabik, 2002, pp. 261-273]. However, their approach did not reflect the complexity of the dynamics of economic phenomena. A significant stage was a suggestion, submitted in 1975, by R. May and J.R. Beddington on the use of chaos theory in economics [May & Beddington, 1975, p. 35]. This resulted in intensified research in this area, which led to the formation of an innumerable quantity of new economic models based on the assumptions of chaos theory. In chaos theory, it is assumed that the complexity of the phenomena may be a cause of considerably different in time systems' behaviour (exponential divergence of trajectories in phase space) with similar phase portraits in the initial phases of the cycle. Dynamic phenomena may therefore be described by dynamic models whose irregularity depends on the degree of their nonlinearity [Siemieniuk, 2001].

This paper is aimed at presenting the basic issues concerning the analysis of experimental data, as well as the properties of deterministic chaos in relation to economic processes.

1. Analysis of the experimental data

Takens (according to [Schuster, 1989; Takens, 1981]) proved that following the disappearance of transient effects, the attractor can be reconstructed by means of measurement of a single component of a vector describing the dynamic system. The application of Takens theorem – in connection with possibilities of modern measuring and computing techniques – enables an analysis of chaotic processes on the basis of experimental data. The analysis of the measuring signal allows determination of several characteristics of dynamic systems including, correlation dimension, Kolmogorov entropy and Lyapunov exponents.

For the measured data in the form of time series determination of all Lyapunov exponents, it is not possible. It is, however, possible to determine the value of the largest Lyapunov exponent. In this case, on the attractor immersed in D dimensional space two points situated at a distance of at least one orbiting period one from another, are selected. The distance between the points is $L(\tau_j)$. The distance of the selected points after the passage of some evolution time is calculated. New distance of the pair of points is $L'(\tau_{j+1})$. The largest Lyapunov exponent is calculated according to the formula [Peters E.E., 1997]:

$$L_1 = \frac{1}{\tau} \sum_{j=1}^{m} \log_2 \frac{L'(\tau_{j+1})}{L(\tau_j)} \tag{1}$$

where: m – number of point pairs examined, τ – time of evolution, $L(\tau_j)$ – the distance between the points, $L'(\tau_{j+1})$ – new distance of the pair of points.

The largest Lyapunov exponent can be determined when such characteristics such as those of attractor as fractal dimension, average orbiting time and time-delay are known. For a long time period, the results of the calculation of L_1 approach stable value, have been an estimation of the largest value of Lyapunov exponent. At least 10^n measuring points and 10^{n-1} orbiting periods [Peters, 1997] are required to determine the largest Lyapunov exponent.

The calculation of the largest Lyapunov exponent is possible only if the fractal dimension of the attractor is known. To determine the fractal dimension of the attractor, value of time-delay (the quantity necessary for the reconstruction of the attractor) is required.

The image of the attractor in n-dimensional space depends upon time–delay τ. When the time-delay is too small, the attractor gets flattened, which makes further analysis of its structure impossible. The selection of time–delay value is of great significance in the analysis of the attractor properties. Therefore the analysis of the experimental data is initiated by determining the time-delay. For that purpose the autocorrelation function is calculated. [Schuster, 1989].

The alternative method of time-delay τ calculation consists in determining of Hurst exponent [Martyn, 1996; Nazarko & Siemieniuk & Mosdorf, 1999]. If the system of numerous degrees of freedom is investigated, it is usually assumed that the changes occurring in the system are of random character (Brownian motion). Hurst established the non-dimensional H exponent, through examining changes of water state in the man-made lakes and dividing the range fluctuations by standard deviation of the observation. This kind of analysis is called the rescaled range analysis R/S [Peters, 1997].

Hurst exponent is determined as follows. The measured data (in the form of samples) are divided into intervals of constant number of points equal to N. For each of the intervals the following series is defined [Peters, 1997, p. 65]:

$$T_i = \sum_{j=1}^{i} (x_j - \overline{x}_N) = \sum_{j=1}^{i} x_j - i \cdot \overline{x}_N \tag{2}$$

where: T_i represents the cumulative deviation for N samples, x_j – the sample value at the time j, $\overline{x}_N$ – the arithmetic average of data for N samples.

For the series (2) in each interval, the R quantity that is called range R = max(*Ti*) – min(*Ti*) and standard deviation S are calculated. R/S characteristic for the whole time series is determined as an average of R/S calculated for all intervals of N length.

The slope of tangent to 1n(R/S) in the function 1n(N) gives the value of H exponent. If N number contains too many measured points the process resembles the random motion (the long-term memory – the memory between succeeding intervals disappears). In this case the slope of a curve changes. For the signals of stochastic character H=0.5 [Martyn, 1996; Peters, 1997]. Border point N* between area where the H>0.5 and area where H=0.5 corresponds with the boundary of the natural period of a physical system. N* quantity enables determining of time-delay τ necessary for attractor reconstruction; τ is calculated from the relation: $\tau = N^{*}/d$.

Values shuffled logarithmic rate of return can be found in the literature [Czarnecki &Grech, 2010].

The trajectories of the chaotic system in the phase space do not form any single geometrical object such as a circle or torus, but rather form objects called strange attractors of the structure resembling the one of a fractal. One of the essential characteristics of fractals is their dimension [Kudrewicz, 1993; Parker & Chua, 1987; Peters, 1997; Schuster, 1987]. For experimental data, the correlation dimension D is defined by the following expression [Chorafas, 1994]:

$$D = \lim_{l \to 0} \frac{1}{\ln l} \ln C(l) \tag{3}$$

where: *C(l)* – of the curve.

In an analysis of the properties of dynamic system the notion of correlation entropy in the following form is introduced [Schuster, 1989]:

$$K = -\lim_{l \to 0} \lim_{n \to \infty} \frac{1}{N} \ln \sum_{i_0 \dots i_{n-1}} P_{i_o \dots i_{n-1}} \tag{4}$$

where: $\sum P_{i_o \dots i_{n-1}}$ quantity is total probability of finding of the system trajectory in the determined area of the phase space. *K* is a lower limit of the Kolmogorov entropy. Condition *K*>0 is sufficient for the existence of chaos in the system investigated. *K* and *D* are interrelated as follows [Schuster, 1989]:

$$\lim_{l \to 0} \lim_{n \to \infty} \ln C^d(l) = D \ln l + MK \tag{5}$$

where: *C(l)* – of the curve.

Fractal dimension is calculated by determining the value of the slope of the regression line crossing a middle region of the curve *C(l)*.

For the stochastic signal the fractal dimension increases with the increase of the embedding dimension. If the signal examined is of deterministic chaos character, then the value of a slope of the regression line approaches constant value *D*. The value determines the correlation dimension of the attractor investigated. Calculation of the correlation dimension is being done for the embedding dimension $d>2D+1$, where *D* is the correlation dimension of the attractor considered [Nazarko & Siemieniuk & Mosdorf, 1999].

In the graph of $\ln C^d(l)$ as a function of $\ln(l)$, correlation entropy, *K*, can be calculated as the difference between curves $\ln C^{d-1} - \ln C^d$ for $\ln(l) \to 0$. With the embedding dimension increase, value of *K* entropy should approach constant value.

2. Fractal analysis of Warsaw Stock Exchange Index

The basic indicator of share prices on the Warsaw Stock Exchange is WIG (Warsaw Stock Exchange Index) which includes all companies on the main stock market.

The WIG index quotations did not take place regularly in the initial phase of the Polish stock exchange development. In order to perform the fractal analysis, a set of data showing WIG changes in particular days needed to be created. It was assumed that for days when WIG quotations were not calculated, WIG value equals the value of the last quotation. Time series set in this way was used to build time runs showing changes of WIG values in time periods of constant number of days. Results of fractal analysis are presented in the Table 1, 2, 3.

Table 1. Hurst exponent and cycle length for WIG (16.04.1991-16.11.2014, weekly data)

Index in question	Hurst exponent	Cycle length for WIG
WIG (based on a series of increments)	0.69	52.7

Source: Own calculations.

Table 2. Fractal dimension for WIG (16.04.1991-16.11.2014, weekly data)

Index in question	Fractal dimension	Number of variables
WIG (based on a series of increments)	2.37	3

Source: Own calculations.

Table 3. Largest Lyapunow exponent for WIG (16.04.1991-16.11.2014, weekly data)

Index in question	Largest Lyapunov	Cycle in months 1/ Largest Lyapunov Time of information loss
WIG (based on a series of increments)	0.0041 bit/7days	(1/0.0041)*7/30=51.9

Source: Own calculations.

Lyapunow exponent describes the rate of information loss in the system. For WIG index, the system loses memory after about 51.9 months. If in the beginning our information about the system is 1 bit, then after 51.9 months we lose the whole information about the system. We can also say that the system loses memory about initial conditions.

Results of Lyapunow exponent calculations for Polish stock exchange are burdened with errors because of small number of measured data. However, comparison of result calculations of Lyapunov exponent to other capital markets [Peters, 1997] indicates the comparability of obtained results (Table 3).

3. The theory of deterministic chaos and the foreign exchange market

The foreign exchange market are "all foreign exchange transactions, and thus the buying and selling one currency for another" [www1]. The abovementioned market is mainly characterized by high liquidity of assets. There are three main groups of entities in the market, i.e. investors involved in speculation, companies engaged in cross-border trade and central banks. According to the elementary laws of economics, exchange rates are a simple, strictly deterministic function of the demand and supply of currencies. What prevents a clear forecast of the state of the system, in this case the exchange rate, are factors affecting supply and demand. The main problem in the functioning of the currency market is defining all decisive elements determining buying or selling the currency. These elements are extremely diverse. In practice, the following groups can be distinguished: global, national and local.

However, it should be noted that this division is extremely conventional and flexible, and the individual factors are not completely separate and can affect each other with different intensity and at different direction (may intensify or weaken other factors). The global factors include, above all else, the overall condition of the global economy. This is particularly evident when the economic situation is weakening. Investors usually turn to more stable currencies, endowed with much more confidence, as the US dollar, the Swiss franc or Japanese yen, while selling off currencies of developing countries. The national factors include, among others, indebtedness of the country and its debt service coverage, general confidence in the currency, the economic situation of the country. These are the factors that affect the exchange rate in the long term, allowing the trends of changes in the long term to be determined. There are also deterministic factors, it is possible to identify their condition and to forecast their impact on the future exchange rate. An important issue is the fact that these factors cannot be considered in absolute terms. The foreign exchange market is an international market, and exchange rates are presented as a ratio of one currency to another, therefore all national rates should be considered relatively, compared to the same rates of other countries.

The last group are the local factors. They are mainly short-term and conscious interventions of key players in the market, resulting in relatively large fluctuations in the exchange rate of a specific currency in a relatively short period. These may be planned and predictable as central bank interventions, selling off or buying out local currency to improve the situation on the internal market or hidden, as currency speculation on the futures and derivatives by major investors or their groups. An example of the latter phenomenon may be a significant fall in the zloty in 2008. The decrease also resulted from another, very important reason, i.e. risk propensity of investors. This characteristic is often essential for a fairly significant change in exchange rates. However, it is difficult to measure and depends on a huge number of secondary factors. The level of investors' confidence is affected by e.g. statements of politicians and economists or created rumours about the possible financial difficulties of a particular entity. Their real significance cannot be clearly defined, they often have an opposite influence to expectations, and other force than a forecast. An example of this might be an increase in the dollar rate despite a reduction of the creditworthiness rate of the United States in August 2011 by one of the three rating agencies.

It should be noted that the factors listed above are only a part of the whole broad spectrum of elements affecting exchange rates. Consequently, many economists and investors ask themselves whether there is any algorithm allowing to predict carefully the behaviour of exchange rates in the future. In the search for an answer some experts reached for chaos theory. Moreover, they indicate that exchange rate fluctuations are a non-linear and multidimensional process, and thus possible to be described by the use of chaotic dynamics. Their main aim is to show that markets do not behave on the principle of "a random walk", which is totally random, but that it is possible to define mathematical equations describing the foreign exchange market. Also models possible to define mathematically are created [www2]. However, these are theoretical models that do not include many variables that occur in reality. Due to their simplification, they use synthetic strings of data, without noise, which was mentioned previously. For such systems it is possible to determine the Lyapunov exponents which are a strict determinant of the chaotic nature of a system [www3]. However, the complexity of models is growing very rapidly in comparison with the increase in the number of variables taken into account, and their behaviour becomes virtually indistinguishable from the random one, given computing means. Despite this, it is assumed that the foreign exchange market should not be treated as completely random, for which no price, no event or no change are interlinked [Siemieniuk & Siemieniuk, 2012].

Moreover, a significant part of the models takes into account only the endogenous, internal factors, relying mainly on statistics and historical data. This is actually a gross oversimplification, allowing more accurate analysis. However, they do not provide the complete picture, because, as mentioned earlier, the foreign exchange market is also influenced by external factors, e.g. political or psychological ones. Again referring to the popular saying, it is impossible to predict where and when a butterfly flaps its wings. Exchange rates yield to market trends. This means that if the price level begins to follow in a particular direction, it is likely that it will continue until such time as something important changes the overall picture of the market. However, the exchange rate, not only on the Polish market, never moves in a straight line [Siemieniuk & Siemieniuk, 2012].

To understand why exchange rates move irregularly, and not in a straight line, it is needed to refer to the previously described chaos theory. In the market the key to success is to recognize the price formations, and they are exactly nonlinear systems that are created on the one-minute chart and the daily chart. It is very difficult to recognize such a system in a short period, but when a longer period is taken into account, the observed system begins to take shape and certain trends are noticeable. In the foreign exchange market, a part of chaos theory is fractal geometry. Briefly speaking, it is assumed that the system that presents the specified 15 minutes chart will be in principle repeated in an hourly, daily and weekly chart. It is connected with a certain trend, and the trend is repeated systematically, called a development trend as the property of a particular time series showing a consistent decrease or increase in its value over a specified period. Thus, three types of trends are distinguished, namely an upward trend (bull), downtrend (bear) and sideways trend [Siemieniuk & Siemieniuk, 2012].

The first trend, called boom, represents the exchange rate which changes with an upward trend, so local minima are higher and higher in the chart, while price increases, as shown in a below presented figure (Fig. 1).

Fig. 1. The upward trend (bull market) on the example of GBP/JPY, daily chart

Source: [www4].

The downward trend on the other hand shows the exchange rate fluctuating with a downward trend. In this case local maxima is lower and lower and the price falls. The above situation is presented in Figure 2.

Fig. 2. The downward trend (bear market) on the example of GBP/JPY, weekly chart

Source: [www4].

The sideways trend is referred to as consolidation. In this trend the rate of exchange does not increase significantly, nor has a downward trend. This means that the exchange rate varies but in such a way that it is within a certain range. The above situation is presented in Figure 3.

Fig. 3. Sideways trend on the example of GBP/JPY, 1 hour chart

Source: [www4].

Exchange rates on the market yield to some trends, namely the level of price begins to move in a particular direction, so there is a high probability that this state of affairs will continue until the moment when something significant changes the overall situation of the market.

Conclusions

As it has already been mentioned, in the theory of deterministic chaos, it is assumed that the complexity of certain phenomena can cause strongly different behaviour of the systems in the period, so this case pertains to exponential divergence of trajectories in phase space, with similar phase portraits in the initial phases of the cycle.

This theory can be used in predicting exchange rates. However, the exchange rate on the Polish market never moves in a straight line. Then it is possible to recognize price patterns (non-linear systems), which are formed on one minute and intraday charts. It is very difficult to recognize such a system in a short period, but when a broader period is taken into account, the observed system begins to take shape and certain trends in the movements of exchange rates are noticeable. For this purpose previously described fractal geometry is used.

The paper has presented the assumptions of the theory of chaos and the possibility of applying it to describe and forecast the currency markets as well as the stock market. The use of complex mathematics and other analytical tools allows us to create approximate expectations about the state of the markets in the relatively near future, however, with increasing length of the forecast, its accuracy approaches zero, because of, among other things, the "butterfly effect".

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CHAOS DETERMINISTYCZNY W MODELOWANIU PROCESÓW EKONOMICZNYCH

Streszczenie: Modelowanie procesów ekonomicznych jest przedmiotem licznych badań i analiz. Wykorzystuje się w nich różne metody i narzędzia badawcze. Próby opisu funkcjonowania zjawisk ekonomicznych podejmowane są w ramach wielu dyscyplin naukowych, np. ekonomii, matematyki, psychologii. Rozmaitość stosowanych teorii i metod znajduje swój wyraz w różnorodności uzyskiwanych wyników i prognoz. W celu przewidywania przyszłego zachowania się rynku akcji czy rynku walut konstruowane są rozmaite modele, które nigdy nie dają pełnej pewności sukcesu i są zwykle obarczone ryzykiem inwestycyjnym. Jedną z nowszych koncepcji modelowania procesów ekonomicznych, takich jak rynek akcji, rynek walut, jest teoria chaosu deterministycznego. Stanowi ona próbę odejścia od idei efektywności rynków kapitałowych czy rynków walutowych w stronę bardziej uniwersalnego widzenia mechanizmów rządzących nimi. Cechy charakterystyczne, stany nierównowagi oraz mechanizm sprzężenia zwrotnego w wymiarze czasowym znajdują swój wyraz w opisie za pomocą dynamicznych systemów nieliniowych. W publikacji omówiono wybrane zagadnienia wykorzystania teorii chaosu deterministycznego do opisu funkcjonowania wybranych zjawisk ekonomicznych. Publikacja zawiera zagadnienia dotyczące istoty chaosu deterministycznego, wybrane przykłady wykorzystania teorii chaosu do opisu zjawisk ekonomicznych ze szczególnym uwzględnieniem rynku kapitałowego i rynku walut.

Słowa kluczowe: chaos deterministyczny, rynek kapitałowy, rynek walut.