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VISUALISATIONS OF THE RISK INVESTMENT VALUATION AND THE LEVEL OF INVENTORY CONTROL USING THE GEOGEBRA SOFTWARE

Summary: The development of information technology forces the usage of software tools in the analysis and visualisation of risk in various fields including economics, finance, management. The graphic presentation of analysis results as well as various relationships contributes to their better understanding. Modern computer software allows for showing dynamics of various decision problems. The aim of the paper is to present the dynamic visualisations of risk analysis in selected fields using the GeoGebra software.

Keywords: visualisation in GeoGebra, risk, portfolio theory, omega ratio, inventory control.

Introduction

Nowadays visualization is not only a form of static presentations, but also tool of analysis, selection and perception of phenomena, facts and trends presented using graphical methods. Visualization can also be understood as the processing of complex relationships into the form which is readable for the customer (using the mechanism of perception) with the use of graphic elements [Dudycz, 1998]. Empirical studies revealed that people perceive up to 87% of the information by sight, 10% of the information by hearing, and only 3% of the information by other senses [Niemann, de Mori and Hanrieder, 1994]. The importance of graphic elements for cognition, understanding and remembering of contents reflects the proverb “A picture is worth a thousand words”. The aim of the graphics and animation is, among others, to complement the text, and create positive associations related to the presented content. The verbal information enriched with appropriate graphics, are better understood and remembered than the same message presented only in spoken form [Kozak and Łaguna, 2014]. The aim of the paper is to present the dynamic visualisations of risk analysis in investment and logistics decisions using the GeoGebra software.

GeoGebra is "(…) a multi-platform mathematics software that gives everyone the chance to experience the extraordinary insights that math makes possible" [www 1]. It combines algebra and geometry and allow for dynamic visualisation of concepts with mathematical background.

1. Risk and its valuation

In making their decisions, the decision-makers are interested not only in future benefits but also in risk relating to that decision. The notion of risk is ambiguous and complex, and its causes can be traced to the uncertainty of external factors (environment) and the uncertainty resulting from the decision strategy. Thus, although the risk is a widely known concept and it is taken into account in the decision making process, its definition and interpretation is ambiguous. Objective as well as subjective aspect of risk is expressed in the way it is measured. If the causes of risk are objective, then its measure is based on random states of nature and probability distributions of variables which are of course objective. In contrast, a reflection of the subjective nature of risk is the analysis based on the utility theory and preference relationship.

In the literature, risk measures are grouped in three categories [Kopańska--Bródka, 1999]: parametric measures (based on the parameters and characteristics of the probability distribution of random variables, e.g. a random rates of return), nonparametric measures (related to the assumed level of "safety" or "ruin"), and criteria which are the basis for the order relations (outranking relations, domination, ordering and comparison of decision alternatives in terms of the value of criterion).

For the first group of risk measures we will present an example of portfolio analysis. In this approach risk is measured by variance or standard deviation of the rate of return. An example of the nonparametric measure of risk is the valuation of the probability of product shortage in the inventory control model. In the third group of risk measures we will present an example of the outranking relation based on the performance measure (omega ratio). Decision alternative with lower value of the omega ratio is seen as more risky alternative.

2. Examples of risk valuation in investment decisions

In this section we will present two examples of visualisation in valuation of risk in investment decisions.

Portfolio analysis

In the classical portfolio theory the selection of the optimal investment decision is based on two characteristics of the investment: gain and risk which depend on gain and risk of all components of portfolio. Usually gain of stock is measured by the expected rate of return and risk by the variance or the standard deviation of rates of return. To illustrate some dependences between shares of stocks, expected rates of return and standard deviations we will focus on portfolios consisting of at most three stocks. All possible portfolios can be visualized in two different planes: plane of shares of stocks and risk-gain plane. Figure 1 shows a set of exemplary portfolios consisting of three stocks A, B and C in these two planes. For all stocks we generated sets of rates of return assuming in all cases a normal distribution with following parameters: A~N(0.01, 0.03), B~N(0.06, 0.05) and C~N(0.02, 0.04).

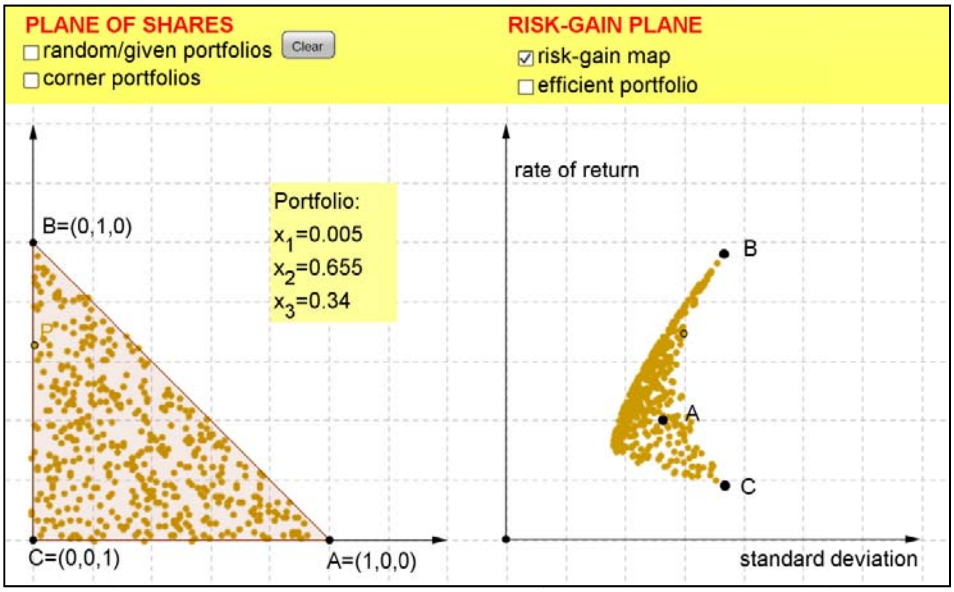

Fig. 1. Random three-stocks portfolios in plane of shares and risk-gain plane

Source: Own elaboration in GeoGebra.

Theoretically, the optimal portfolio of stocks is that which simultaneously maximizes the expected rate of return and minimizes the standard deviation. Usually such criteria are contradicting (there is no “the best of the best” portfolio). Therefore the investor is interested in the subset of all portfolios called the efficient frontier. A portfolio is efficient if none other gives either a higher expected rate of return and the same standard deviation of rates of return or a lower standard deviation of rates of return and the same expected rate of return [Haugen, 1996; Elton and Gruber, 1998]. Any of the efficient portfolio can be determined by solving the following optimization model:

$$
\begin{aligned}
& V_P^2 \to \min \\
& E_P \geq R_0 \\
& \sum_{i=1}^{N} x_i = 1 \\
& x_1, \ldots, x_N \geq 0
\end{aligned}
\tag{1}
$$

where:

V_P^2 – variance of rates of return of portfolio,
E_P – expected rate of return of portfolio,
R_0 – desired rate of return of portfolio,
x_i – share of i-th stock in portfolio,
N – number of all stocks.

The efficient frontier is received by changing the R_0 level in the interval $\langle \min(R_i), \max(R_i) \rangle$ and solving appropriate models. The efficient portfolios state the set of potential optimal decisions for the Mean-Variance criterion. Determining all of the efficient portfolios could be exhausting, but not all of them are necessary. Any set of efficient portfolios can be described in terms of a smaller set of corner portfolios. These portfolios differ in exactly one stock. The algorithm for finding corner portfolio is describe for example in [Kopańska--Bródka (ed.), 2004; www 2]. Any of the efficient portfolio can be determined as a linear combination of two adjacent corner portfolios. Therefore the examination of efficient frontier can be reduced to the finite set of corner portfolios.

In Figure 2 portfolios B, P_1, P_2 and MV are corner portfolios. Portfolio B is a stock with the highest expected rate of return, portfolio P_1 consists of two stocks (B and A), portfolio P2 consists of two stocks (A and C) while the last portfolio MV is the global minimum risk portfolio containing A and C stocks.

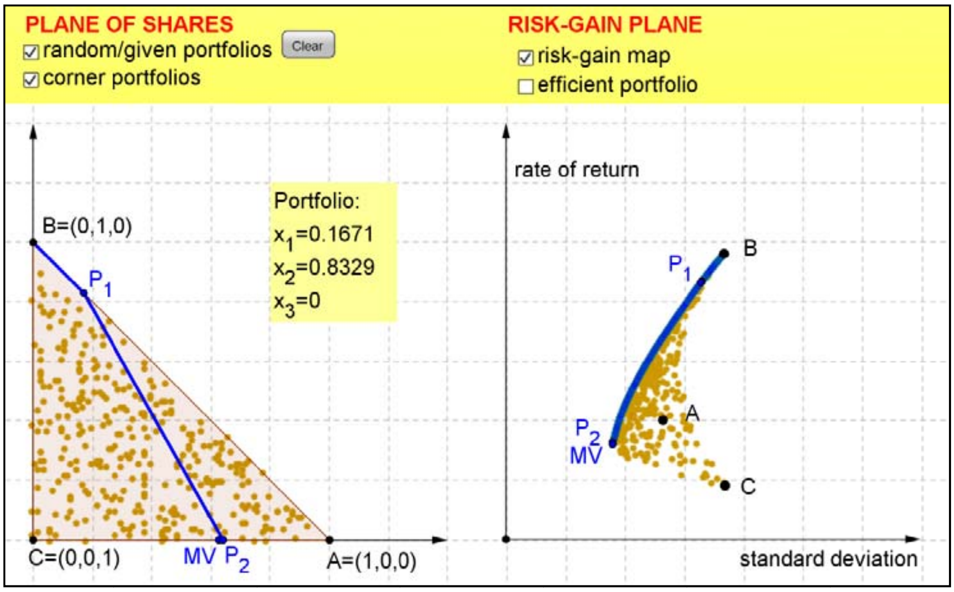

Fig. 2. Corner portfolios and efficient frontier in plane of shares and risk-gain plane (first set of data)

Source: Own elaboration in GeoGebra.

For three considered stocks an investor can analyze another sets of realizations of random distributions of rates of return. Figure 3 presents corner portfolios B, P_1 and MV. Portfolio B consists only of stock B which has the highest expected rate of return, portfolio P_1 consists of two stocks (B and C) Portfolio MV, as above, is the global minimum risk portfolio but in this case it contains all three stocks A, B and C.

In Figures 2 and 3 we can see the differences in the obtained estimates of gain and risk of portfolios consisting of shares for which returns in both cases were generated with the same distributions.

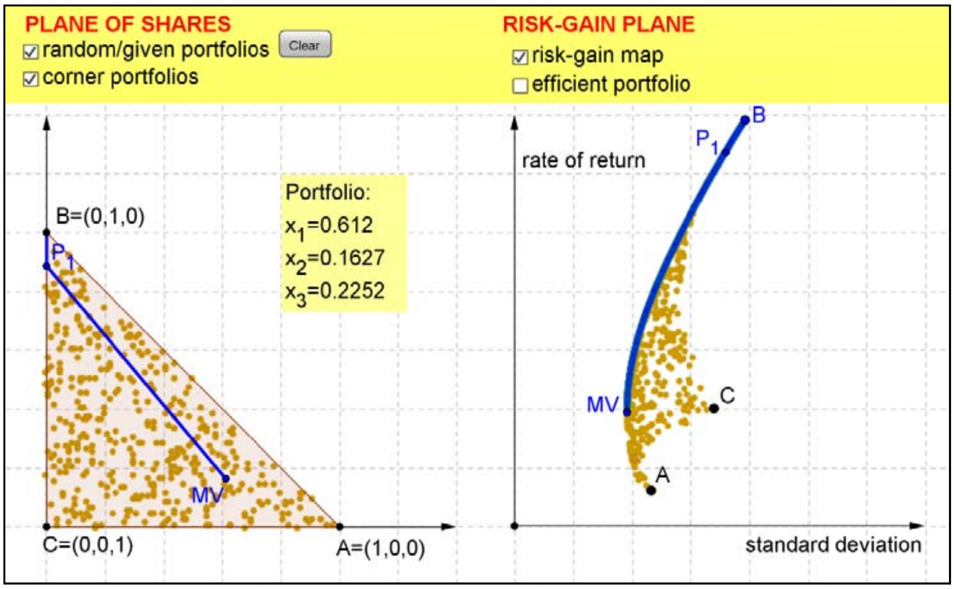

Fig. 3. Corner portfolios and efficient frontier in plane of shares and risk-gain plane (second set of data)

Source: Own elaboration in GeoGebra.

Omega ratio

The motivation for Keating and Shadwick [2001], the authors of omega function and omega ratio, was the observation that moments of the distribution (mean and variance) do not fully describe the distribution of a random rate of return. They proposed the measure which takes into account all information about distribution of the random variable. The form of the omega function also gives the investor the opportunity to take into account the preferences expressed by the threshold level (e.g. acceptable rate of return) in respect of which the investment results are judged as desired (values higher than the threshold) or undesired (values lower than the threshold). Omega function for a fixed value of argument (threshold level) is the performance measure of a considered investment. It is a ratio of the expected gains above the threshold level to the expected losses below the threshold.

The omega function is defined as a ratio of probability weighted gains to probability weighted losses, relative to a threshold level:

$$\Omega(L) = \frac{I_2(L)}{I_1(L)} = \frac{\int_L^b (1 - F(t))dt}{\int_a^L F(t)dt} \tag{2}$$

where:
F – cumulative distribution with non-trivial domain (a, b) and finite mean μ,
L – threshold level (benchmark) selected by an investor and $L \in (a,b)$.

Figure 4 shows the visualisation (geometric interpretation) of the omega function (upper part) for the normal distribution N(2.1, 1.2) shown in the lower part of figure. The omega function is also defined as the ratio of area $I_2(L)$ to area $I_1(L)$, where $I_2(L)$ is a light shaded (light gray) area above the distribution representing gains and $I_1(L)$ is a shaded area (gray) below the distribution representing losses. Researchers have observed many properties of this measure [Michalska and Kopańska-Bródka, 2015]. The omega function $\Omega(L)$ is continuous and decreasing on its domain, taking values from the interval $(0, +\infty)$ and $\Omega(\mu) = 1$.

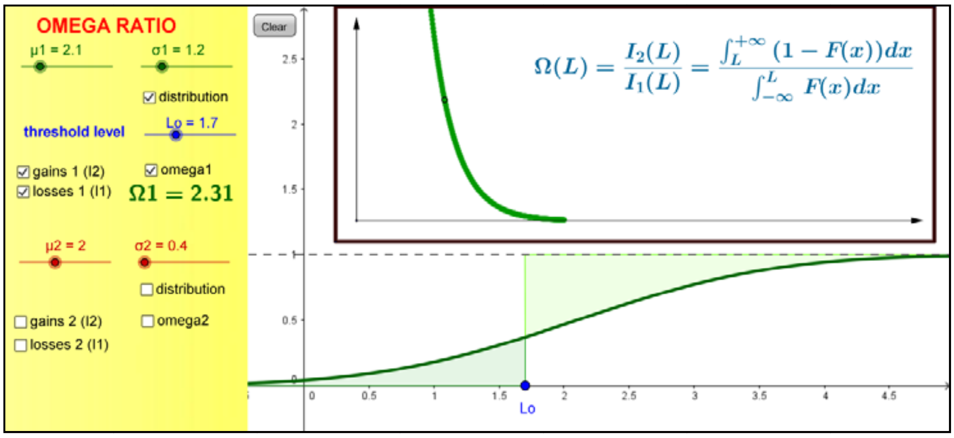

Fig. 4. Omega function for normal distribution N(2.1, 1.2)

Source: Own elaboration in GeoGebra.

The omega function and the omega ratio as well as the stochastic dominance can serve as a tool for comparing two or more investments defined by its distributions [Michalska and Dudzińska-Baryła, (in print); Trzaskalik, Trzpiot, Zaraś, 1998]. Figure 5 presents the omega functions of two investments described by normal distributions N(2.1, 1.2) and N(2, 0.4). The visualisation of the both omega functions allows for valuation of risk of this two alternatives. An investment with lower value of the omega ratio is seen as more risky than an investment with higher value of the omega ratio. In our example, if the decision--maker's (investor's) threshold level is below 1.95 (the same units as for mean) then the investment with distribution N(2.1, 1.2) is preferred to the other investment. The opposite relation holds for the threshold level above 1.95. The visualisation of both distributions allows us also to identify that distribution N(2.1, 1.2) dominates N(2, 0.4) by the SISD (Second Inverse Stochastic Dominance).

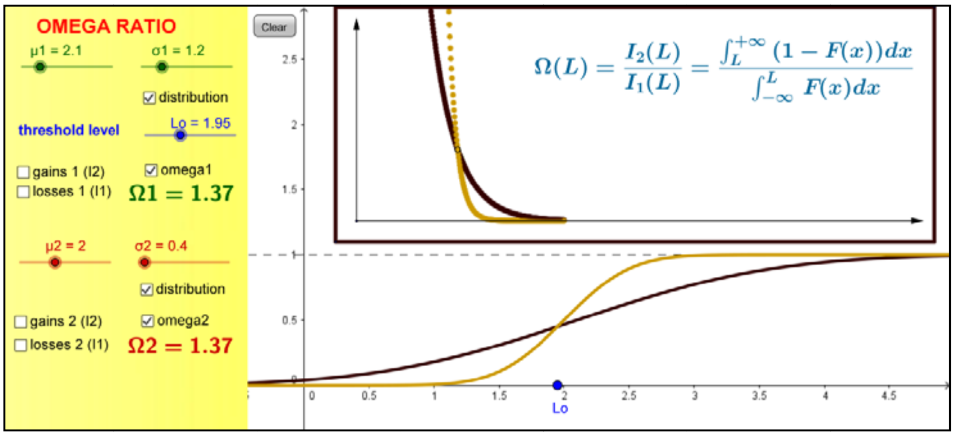

Fig. 5. Comparison of two investments based on the omega function (case 1)

Source: Own elaboration in GeoGebra.

In some cases the omega functions do not cross, and then one investment is preferred to the other for any threshold level. Such situation is illustrated in Figure 6, where investment with the distribution N(2.1, 0.9) is preferred to the investment with the distribution N(1.7, 0.7). We can also notice that distribution N(2.1, 0.9) dominates N(1.7, 0.7) by the FSD (First Stochastic Dominance).

The use of sliders (which is a standard object in GeoGebra allowing for dynamic change of value of parameter) for the parameters of random distribution and threshold level allows to analyze the impact of changes in these parameters on the value of the omega ratio. Generally, the omega function and omega ratio can be used in the valuation of risk of decision alternatives (investments) for any probability distribution function, e.g. uniform or normal distribution [Michalska and Dudzińska-Baryła, in print].

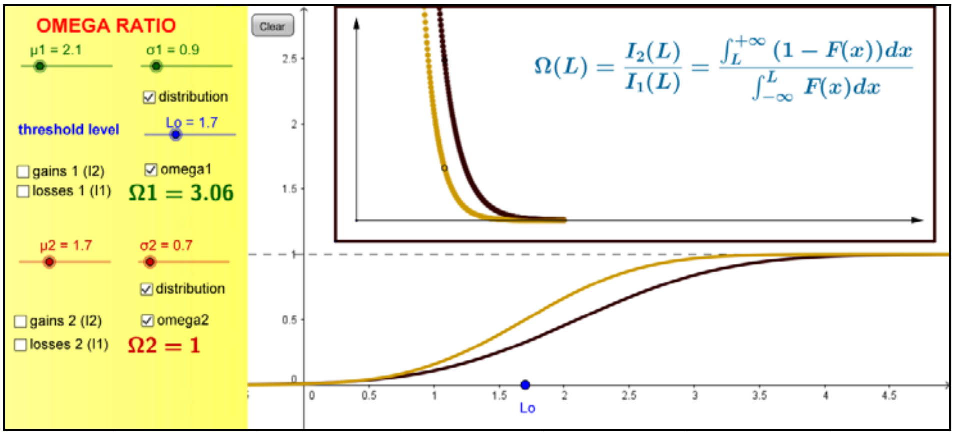

Fig. 6. Comparison of two investments based on the omega function (case 2)

Source: Own elaboration in GeoGebra.

3. Examples of risk valuation in logistics decisions

Analysis in the field of the inventory control and its visualisations contribute to the efficient management of inventory, which is one of the most important elements of the business. Inventory is usually defined as resources of goods used in production process (raw materials and production in progress), in auxiliary activities (materials for maintenance, repair and operations) and in the customer services (finished products and spare parts). Proper inventory control ensures the continuity of business processes, production and selling [Bozarth and Handfield, 2007]. There are two main model of inventory control: with continuous review and with periodic review. In this article we explore the model with periodic review, in which a reorder decision is permitted to occur only at fixed intervals of time (e.g. every week or every 5 weeks). Figure 7 shows the visualization of the inventory control process with periodic review for independent demand.

The visualisation prepared in GeoGebra shows a graph of inventory level changing in time, for random (normally distributed) demand. The use of sliders for the review period, maximum inventory level or parameters of random distribution of demand allows to observe the impact of changes in these parameters on the level of inventory in each period, and to notice possible shortages (points below the horizontal axis). In addition, the use of a random number generator allows to analyze multiple variants of the considered inventory control problem.

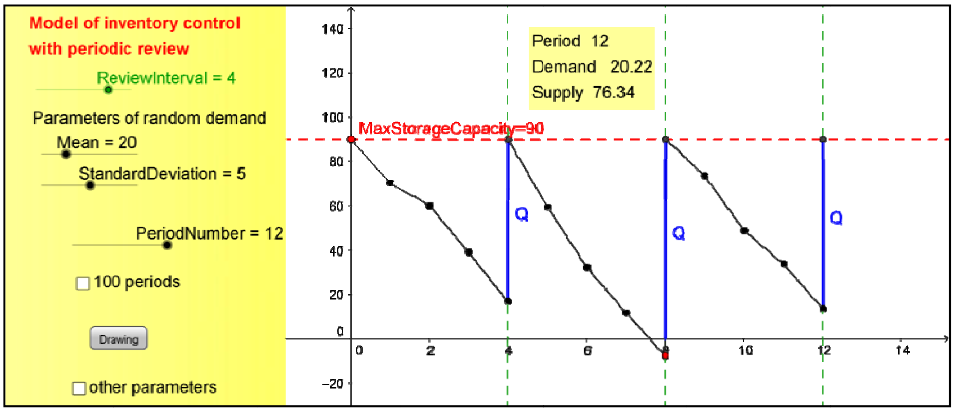

Fig. 7. Periodic review model of inventory control (for twelve periods)

Source: Own elaboration in GeoGebra.

Our visualisation is very useful in the valuation of risk of product shortage. Let us assume that storage capacity equals 90 and demand is normally distributed with mean 20 and standard deviation 5. If we consider e.g. 100 periods with review every 4 periods the probability of product shortage for some realisations of random demand may equal 0.07 with the average size of shortage at 5.35 level (see Figure 8). After generating a new set of realisations of random demand the probability of product shortage may be as small as 0.01 with the average size of shortage at 0.22 level (see Figure 9).

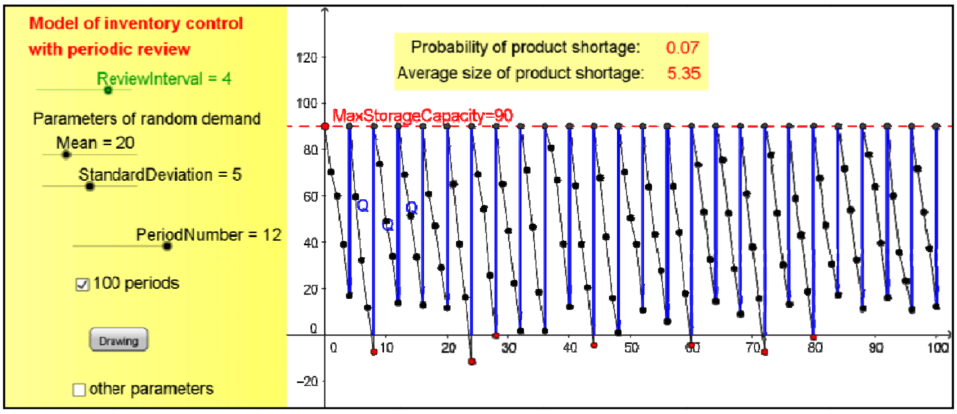

Fig. 8. Periodic review model of inventory control for 100 periods

Source: Own elaboration in GeoGebra.

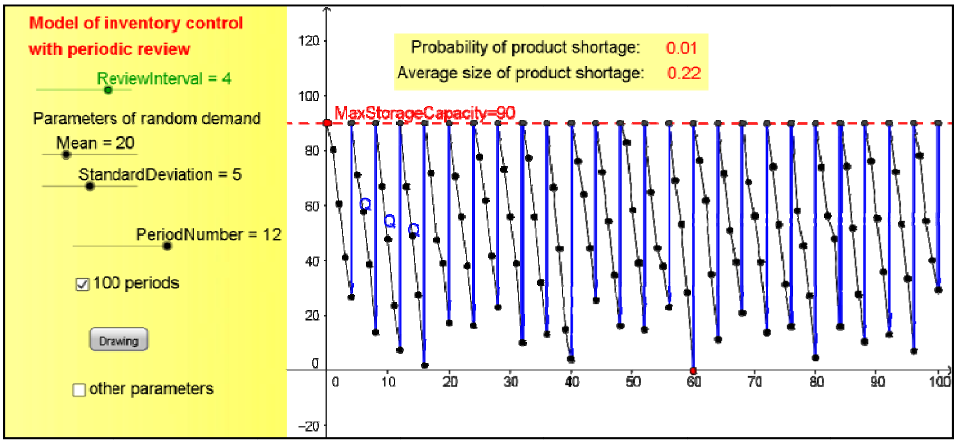

Fig. 9. Probability and average size of product shortage

Source: Own elaboration in GeoGebra.

Such a simple analysis as above can be useful for the management of quality of service. In order to increase the quality of service the manger should increase the storage capacity, which in turn will result in higher costs of storage or decrease the interval between reviews, which in turn will increase cost of replenishment (ordering). There is always a trade-off between cost and quality of service.

Conclusions

Modern software tools are not only used for data analysis, but they also allow to present the results of these analyzes in the form of various summaries or graphs. Processing of information into an image helps in better understanding of the dependences, which just numbers cannot express in such an intuitive way. Visualization used to expose the dynamics of phenomena is therefore an important element which supports decision-making. The examples of issues from different areas of human activity (investment, inventory management), presented in this paper, indicate the broad possibilities of using GeoGebra in a dynamic risk analysis. The user-friendly environment of GeoGebra makes for creating their own presentations, dynamic visualizations and even animation useful in decision-making.

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[www 2] http://www.stanford.edu/~wfsharpe/mia/opt/mia_opt3.htm (access: 15.09.2015).

WIZUALIZACJE W OCENIE RYZYKA INWESTYCYJNEGO I STEROWANIU POZIOMEM ZAPASÓW Z WYKORZYSTANIEM PROGRAMU GEOGEBRA

Streszczenie: Rozwój technologii informatycznych wymusza korzystanie z programów komputerowych w zakresie analizy i wizualizacji ryzyka w różnych dziedzinach, w tym ekonomii, finansach, zarządzaniu. Graficzne przedstawienie wyników analizy, jak również różnych zależności, przyczynia się do ich lepszego zrozumienia. Nowoczesne oprogramowanie komputerowe pozwala na pokazywanie dynamiki różnych problemów decyzyjnych. Celem artykułu jest przedstawienie wybranych dynamicznych wizualizacji ryzyka utworzonych z wykorzystaniem programu GeoGebra.

Słowa kluczowe: wizualizacja w programie GeoGebra, ryzyko, teoria portfela, wskaźnik omega, sterowanie zapasami.