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SOME PROPERTIES OF COMPETITIVE EQUILIBRIA AND STABLE MATCHINGS IN A GALE-SHAPLEY MARKET MODEL

Summary: In the paper we study some properties of competitive equilibria in the market model of Gale-Shapley type. We introduce a formal definition of equilibrium in this model without assumption about zero pricing of unassigned goods. We also prove that competitive equilibrium allocations in this model are strongly stable matchings in the Gale-Shapley sense (and vice versa).

Keywords: Gale-Shapley market model, Competitive equilibrium, Stable matching, Indivisible good, Unit demand market model.

Introduction

Market models with finite number of indivisible goods and unit demand became very popular since Gale [1960] presented his model of buying *n* houses by *n* buyers [Gale, 1960, Ch. V, § 6] and Shapley and Shubik [1971/72] developed their theory of „assignment games". The Gale and Shapley [1962] model of college admissions may also be treated as some kind of „market" model with unit demand, yet without prices and money transfers (in the Gale-Shapley model applicants can be interpreted as „buyers" and colleges can be interpreted as „sellers" acting in a „college market" in which „traded" goods are seats in particular colleges).

Nowadays, market models with unit demand are very often used for modelling auction markets, see, e.g. [Andersson and Erlanson, 2013], and hence their importance considerably increased in recent years.

The most important problems concerned with such models are, of course, the problems related to competitive equilibria – existence of equilibria, algorithms for finding the equilibria, properties of equilibria and so on.

A general model of market with unit demand was recently presented by Chen, Deng and Ghosh [2014]. They studied models with general utility functions $u(b, s, p(s))$ and budget constraints $r(b, s)$ (in our notation $u(b, s, p(s))$ is interpreted as utility of good s for a buyer b, when the price of s is $p(s)$, and $r(b, s)$ is the maximal (reservation) price which a buyer b is willing to pay for the good s).

A model which is described in our paper may be treated as some special case of the Chen, Deng and Ghosh's model. We assume that buyers' preferences do not depend on prices and hence their utilities can be described by functions of the type $u(b, s, p(s)) = u(b, s)$. The model is based on the Gale-Shapley college admissions model and it seems that very poor research has been done on studying equilibria in such models till now. The only references we could mention here are the (non-published) paper of Azevedo and Leshno [2014, p. 13, lemma 2] and the author's papers [e.g., Świtalski, 2008, 2014].

In our paper we formally define equilibrium in a Gale-Shapley market model. We do not use the concept of dummy buyer or dummy good, as in typical models [e.g. Mishra and Talman, 2010 or Chen, Deng and Ghosh, 2014]. Instead we introduce the sets of acceptable buyers and acceptable goods. We do not use also the concept of utility, as in Chen, Deng and Ghosh [2014], because we represent preferences by order relations as in the model of Gale and Shapley.

In section 2 we show two simple, but very important properties of such equilibria (maximality and stability with respect to changing the prices of unassigned goods) and show that the traditional assumption of zeroing the prices of unassigned goods may be deleted from the definition with preserving the most important properties of the equilibrium.

In section 3 we show that equilibria allocations in our model can be identified with the so-called strongly stable matchings (here stability is understood as Gale-Shapley stability defined in their paper [1962]).

1. The model

We consider two finite and non-empty sets: B and S. B is a set of buyers and S is a set of sellers. Each seller owns exactly one indivisible object (good) which she wants to sell. We identify sellers with objects they own, hence S may also be interpreted as a set of goods. Goods may be houses, horses, paintings and so on. Each buyer wants to buy no more than one object (so we have unit demand in our market).

We assume that some buyers $b \in B$ may be not acceptable for a given seller $s \in S$ (i.e. S is not ready to enter into a transaction with b) and, similarly, some sellers $s \in S$ may be not acceptable for a given buyer $b \in B$. Hence we consider only pairs (b, s), such that b is acceptable for s and s is acceptable for b. Formally, we assume that a non-empty set of acceptable pairs $F \subset B \times S$ is defined and the sets:

$$F(b) = \{s \in S: (b, s) \in F\},$$
$$F(s) = \{b \in B: (b, s) \in F\}$$

are interpreted as the set of acceptable sellers for a buyer b and the set of acceptable buyers for a seller s, respectively. We assume that for each $b \in B$, $F(b) \neq \emptyset$, and for each $s \in S$, $F(s) \neq \emptyset$.

We assume also that buyers have preferences over the sellers (equivalently – over the objects) and sellers have preferences over the buyers. Preferences are represented by weak orders (transitive and complete relations), i.e. to each agent (buyer b or seller s) an ordered list of agents from the opposite set is assigned, with possible indifferences between the agents.

For example, notation b: $[s\ t]\ u$ will mean that for a buyer b, sellers s and t are indifferent and s is better than u, t is better than u. We will use the notation $s \geq_b t$ meaning that for b, s is better than (or indifferent to) t. Similarly, $s >_b t$ will mean strict preference and $s \approx_b t$ – indifference.

For each buyer b and each seller $s \in F(b)$ we define a reservation price $r(b, s) \geq 0$ (budget constraint) interpreted as the maximal price that b is willing to pay for the object s (in the „college market" the role of reservation price is played by the sum of scores of the applicant b from the disciplines required by the college s).

We assume that preferences of the sellers are determined by reservation prices of the buyers, i.e. for any buyers b and c and a seller s we have

$$b \geq_s c \Leftrightarrow r(b, s) \geq r(c, s) \tag{1.1}$$

A *matching* of buyers with sellers (or an allocation of goods among buyers) is some assignment of buyers to acceptable sellers such that no buyer gets more than one good and no good is sold to more than one buyer. Formally, we have:

(1.2) Definition. *Matching* m is a set of disjoint pairs $m \subset F \subset B \times S$ (two pairs (b, s) and (c, t) are disjoint if $b \neq c$ and $s \neq t$).

If $m \subset B \times S$ is a matching, then $m(b) = s$ or $m(s) = b$ will mean that $(b, s) \in m$ (there is a transaction between buyer b and seller s). If, for a given $b \in B$, there is no $s \in F(b)$ such that $(b, s) \in m$, then we write $m(b) = \varnothing$, and, similarly, $m(s) = \varnothing$ means that there is no $b \in F(s)$ such that $(b, s) \in m$.

In the next section we will need the following notion of maximal matching.

(1.3) Definition. Matching m is *maximal* if there is no matching $n \neq m$ such that $m \subset n \subset F \subset B \times S$.

The next definition is the standard definition of Gale-Shapley stability.

(1.4) Definition. A matching $m \subset F \subset B \times S$ is *stable* if there is no pair $(b, s) \in F \setminus m$ such that $s >_b m(b)$ and $b >_s m(s)$ (we assume that $s >_b \varnothing$ and $b >_s \varnothing$ are always true).

The pair (b, s) satisfying $s >_b m(b)$ and $b >_s m(s)$ in the above definition will be called blocking pair.

We will also use the notion of strongly stable matching which can be defined similarly as in the definition (1.4) with the only difference that the inequality $b >_s m(s)$ is replaced by $b \geq_s m(s)$ (the pair (b, s) satisfying the inequalities $s >_b m(b)$ and $b \geq_s m(s)$ will be called weakly blocking pair).

It is easy to see that any stable (strongly stable) matching is maximal (if no, there would be a pair (b, s) such that $m(b) = \varnothing$ and $m(s) = \varnothing$ and so it would be a blocking (weakly blocking) pair).

Assume now that each seller announces a price $p(s)$ for the object she owns. A sequence of prices $p(s)$ $(s \in S)$ will be called the *price vector p*. For any buyer b and a price vector p we define the set of feasible sellers as

$$F(p, b) = \{ s \in F(b) : r(b, s) \geq p(s) \}.$$

So the seller s is feasible for b if b is ready to buy the object s at the price $p(s)$. We do not exclude here the situation that $F(p, b)$ is empty, i.e. that the prices of all objects are too high for the buyer b.

We define the set of best (maximal) objects for b in the set $F(p, b)$ as:

$$M(p, b) = \{s \in F(p, b) \colon \forall\ t \in F(p, b),\ s \geq_b t\}$$

(and $M(p, b) = \varnothing$, if $F(p, b) = \varnothing$).

2. Equilibria – definition and basic properties

Classical definition of competitive equilibrium (for continuous models) requires that the equilibrium prices are „market clearing" prices, i.e. that for each good, demand for this good equals supply of this good. In the presented model, supply of each good is one and hence, if the number of goods is greater than the number of buyers, then the equality can not hold (because of the unit demand of the buyers). Hence, for the considered model, the definition of competitive equilibrium should be modified. Typical approach, in the theory of unit demand models, is to introduce the assumption of zero pricing of unassigned (undemanded) goods [see, e.g., Mishra and Talman, 2010 and Chen, Deng and Ghosh, 2014].

But in real markets, prices of most of the goods are positive, even if some of them are undemanded (i.e. if there are no buyers which could buy them). From the point of view of continuous models, we have no equilibrium in such situation, yet for the unit demand model it appears that we can define equilibrium even under the assumption of positivity of prices of undemanded goods.

Namely, in our paper we extend the notion of equilibrium for the unit demand models, introducing the so-called weak equilibrium, for which all important properties of standard definition are preserved and in which all prices (even for unassigned <undemanded> goods) may be positive.

We start from the very weak notion of quasi-equilibrium. Then we formally define the notions of unassigned and undesired goods. In a quasi-equilibrium unassigned goods may be interpreted as goods which are undemanded under fixed prices, and undesired goods are those which are undemanded under any price. Using these notions we then define equilibria and weak equilibria and state their properties.

(2.1) Definition. *Quasi-equilibrium* is a pair (m, p) (where $m \subset F \subset B \times S$ is a matching and p – a price vector), satisfying the conditions (for each $b \in B$ and $s \in F(b)$):

$$(1)\quad F(p, b) \neq \emptyset \quad \Rightarrow \quad \exists\, s \in M(p, b), (b, s) \in m,$$
$$(2)\quad (b, s) \in m \quad \Rightarrow \quad s \in M(p, b).$$

The first condition says that to any buyer b, for which the set of feasible goods is non-empty, a good $s \in F(b)$ is assigned, which is best for b. The second condition says that the good s which is assigned to b should be the best good for her. In other words, if, for a buyer b, $F(p, b)$ is empty, then no good is assigned to b.

If (m, p) is a quasi-equilibrium, then p will be called quasi-equilibrium prices, and m – quasi-equilibrium allocation. It is easy to see that in a quasi-equilibrium some goods may be not assigned to any buyer and some buyers may not obtain any good (see example (2.5) below).

(2.2) Definition. Let m be a matching. A good s such that there exists b with $(b, s) \in m$ is called *assigned under m*. A good which is not assigned under m is called *unassigned under m*. A buyer b such that there is no s with $(b, s) \in m$ is called *unsatisfied under m*.

(2.3) Definition. Let (m, p) be a quasi-equilibrium and q a price vector. We say that q differs from p only for good s if $q(t) = p(t)$ for all $t \neq s$.

(2.4) Definition. Let (m, p) be a quasi-equilibrium. We say that s is undesired under (m, p), if it is unassigned under m and if (m, q) is a quasi-equilibrium for any price vector q such that q differs from p only for s.

Hence, undesired goods are those which are not demanded under prices p and which will be not demanded even if we change in any way (for example decrease) their prices.

(2.5) Example. Let $B = \{b, c\}$, $S = \{s, t, u\}$, $F = \{(b, s), (b, t), (b, u), (c, s), (c, t)\}$ and preferences of the buyers are: b: $s\ t\ u$, c: $t\ s$, (hence for the buyer b we have $s > t > u$, and for c we have $t > s$). Reservation prices are: $r(b, s) = r(b, t) = r(b, u) = 2$, $r(c, s) = r(c, t) = 1$. Let $p = (2, 2, 2)$ (i.e. $p(s) = p(t) = p(u) = 2$) and $m = \{(b, s)\}$. Then (m, p) is a quasi-equilibrium (we have $F(p, b) = \{s, t, u\}$, $F(p, c) = \varnothing$ and $M(p, b) = \{s\}$, hence the only possible pair in any quasi-equilibrium allocation with prices p is (b, s)). The goods t and u are unassigned under m, but u is undesired under (m, p) (if we take prices $q = (2, 2, q(u))$, i.e. if we take q differing from p only for u, then (m, q) is still a quasi-equilibrium) and t is not undesired under (m, p) (if we take prices $q = (2, 1, 2)$, then (m, q) is not a quasi-equilibrium, because $F(q, c) = \{t\}$ and $M(q, c) = \{t\}$, but $(c, t) \notin m$).

Now we state the standard definition of (competitive) equilibrium. Our definition agree with the definitions presented in [Mishra and Talman, 2010] and in [Chen, Deng and Ghosh, 2014], although the notation here is different and we do not use the concepts of dummy buyer, dummy good and utility function.

(2.6) Definition. A pair (m, p) is called equilibrium, if it is a quasi-equilibrium (i.e. conditions (1) and (2) from the definition (2.1) are satisfied) and if prices of all unassigned goods are zero.

The next two propositions state two important properties of any equilibrium: maximality and stability with respect to changing the prices of unassigned goods.

(2.7) Proposition. If (m, p) is an equilibrium, then m is a maximal matching.

Proof. Assume that m is not maximal. Hence there is a pair $(b, s) \in F$ such that $(b, s) \notin m$, b is unsatisfied and s is unassigned. By the definition (2.6), price of s is zero and hence $s \in F(p, b)$ (s is feasible for b). By (1) from the definition (2.1), there exists $t \in M(p, b) \subset F(p, b)$ such that $(b, t) \in m$. Hence b is satisfied, a contradiction.

Maximality of m means that there is no pair (b, s) of mutually acceptable agents (buyer and seller) such that b is unsatisfied (there is no good assigned to

b) and the good owned by s is not sold (not assigned to any buyer). In special case, if all pairs (b, s) are acceptable, it means that if the number of buyers is greater than the number of sellers, then no good is unassigned, and if the number of sellers is greater than the number of buyers, then no buyer is unsatisfied. Hence the prices p in any equilibrium may, in some sense, be interpreted as „market clearing" prices similarly as in the (neo-) classical theory of competitive (Walrasian) equilibrium.

The next property of equilibrium says that, roughly speaking, for any equilibrium (m, p) we can change the prices of unassigned goods without changing the equilibrium allocation m.

(2.8) Proposition. If (m, p) is an equilibrium, then all unassigned goods under m are undesired under (m, p).

Proof. Take an unassigned good s and let q be a price vector such that q differs from p only for s. It is easy to see that for any buyer b, $F(p, b) \neq \varnothing$ if and only if $F(q, b) \neq \varnothing$ and that $M(p, b) = M(q, b)$ (because price of s is zero and by increasing it we can only decrease the sets $F(p, b)$ without affecting the sets $M(p, b)$). Hence, if the conditions (1) and (2) from the definition (2.1) are satisfied for (m, p), then they are also satisfied for (m, q). Thus, by definition (2.4), s is undesired under (m, p).

Proposition (2.8) is a formal statement of an obvious fact that if some good s is not demanded under price $p(s)$, then it will be not demanded under any higher price $q(s)$.

We can use proposition (2.8) to weaken the notion of equilibrium and to formulate the definition of equilibrium without the assumption of zeroing the prices of unassigned goods.

(2.9) Definition. A pair (m, p) is called weak equilibrium if it is a quasi-equilibrium such that all unassigned goods under m are undesired under (m, p).

Proposition (2.8) implies that each equilibrium is a weak equilibrium. It is also easy to see that if (m, p) is a weak equilibrium, then m is maximal (for a weak equilibrium (m, p) we can decrease all the prices of unassigned goods to zero, thus obtaining an equilibrium (m, q) for which, by proposition (2.7), m is maximal).

Hence weak equilibrium is a reasonable extension of the concept of equilibrium, preserving the property of maximality (thus the weak equilibrium prices can also be interpreted as „market clearing" prices) but without the property of zero pricing of unassigned goods.

3. Relationship with the Gale-Shapley stability

In this section we formulate and prove a result about relationship between the notion of equilibrium in the Gale-Shapley market model and the notion of stable matching in the sense of Gale and Shapley's college admission model [1962]. The result says that a matching m is an equilibrium (weak equilibrium) allocation if and only if it is a strongly stable matching in the sense of Gale and Shapley.

Hence we obtain another important property of any (weak) equilibrium allocation (i.e. the property of being a strongly stable matching) and, on the other hand, a property of any strongly stable matching (being an equilibrium allocation).

The result is the following.

(3.1) Theorem. If (m, p) is an equilibrium (weak equilibrium), then m is a strongly stable matching, and if m is a strongly stable matching, then there exist prices p such that (m, p) is an equilibrium (weak equilibrium).

Proof. First we prove that m is strongly stable for any equilibrium (m, p). Assume that we have an equilibrium (m, p) and m is not strongly stable. Hence there exists a pair $(b, s) \in F \setminus m$ such that $s >_b m(b)$ and $b \geq_s m(s)$. Consider two cases:

(1) $m(s) \neq \varnothing$. Obviously $(m(s), s) \in m$ and, by (2), definition (2.1), we have $s \in M(p, m(s))$. Hence $r(m(s), s) \geq p(s)$. By our initial assumption $b \geq_s m(s)$ and this implies, by (1.1), $r(b, s) \geq r(m(s), s) \geq p(s)$. Hence $s \in F(p, b)$. Thus, by our assumption $s >_b m(b)$, we have $m(b) \notin M(p, b)$, a contradiction with the definition of equilibrium (by (2), definition (2.1) we should have $m(b) \in M(p, b)$).

(2) $m(s) = \varnothing$. In this case s is unassigned and hence $p(s) = 0$. Thus, obviously, $r(b, s) \geq p(s)$, and the same reasoning as previously leads to a contradiction.

Now we prove that for a strongly stable matching m there exist prices p such that (m, p) is an equilibrium.

Assume that m is strongly stable. Define p by $p(s) = r(b, s)$ if there exist $b \in F(s)$ such that $m(s) = b$ and $p(s) = 0$, if there is no such b. Hence prices of all unassigned goods are zero and by definition (2.6) it suffices to show that (m, p) is a quasi-equilibrium. First we prove that condition (1) from the definition (2.1) is satisfied.

Assume that $F(p, b) \neq \varnothing$ for some $b \in B$ and that $(b, s) \notin m$ for all $s \in M(p, b)$. Consider two cases:

(1) there exists $t \in F(b)$ such that $(b, t) \in m$. Hence $b \in F(t)$ and $m(t) = b$. By the definition of p, we have $p(t) = r(b, t)$ and so $t \in F(p, b)$. By our initial assumption $t \notin M(p, b)$. Obviously $M(p, b) \neq \varnothing$. Take any $s \in M(p, b)$. Hence

$r(b, s) \geq p(s)$ (because $s \in M(p, b) \subset F(p, b)$) and $s >_b t$ (because $t \in F(p, b) \setminus M(p, b)$). Consider two subcases:

(a) there exists $c \in F(s)$ such that $m(s) = c$. Thus $p(s) = r(c, s)$ and so $r(b, s) \geq r(c, s)$. Hence, by (1.1), $b \geq_s c = m(s)$ and so (taking into account that $s >_b t = m(b)$) we obtain that (b, s) is a weakly blocking pair, a contradiction with the strong stability of m.

(b) there is no $c \in F(s)$ such that $m(s) = c$. Obviously, (b, s) is also a weakly blocking pair in this case, because $s >_b t = m(b)$ and $b \geq_s \varnothing = m(s)$. Hence, we have also a contradiction here.

We now consider the second case:

(2) there is no $t \in F(b)$ such that $(b, t) \in m$. We have $F(p, b) \neq \varnothing$, hence there exists $s \in F(p, b)$ and $r(b, s) \geq p(s)$. Buyer b is not matched with any $t \in F(b)$, hence, by maximality of m, there exists $c \in F(s)$ such that $(c, s) \in m$. Thus $p(s) = r(c, s)$ and $r(b, s) \geq r(c, s)$ (because $r(b, s) \geq p(s)$). Hence, by (1.1), $b \geq_s c = m(s)$. We have also $s >_b \varnothing = m(b)$, thus m is not strongly stable, a contradiction.

Now we prove that condition (2) from the definition (2.1) is satisfied. Take a pair $(b, s) \in m$ and assume that $s \notin M(p, b)$. We have $(b, s) \in m$ and so $m(s) = b$ and $p(s) = r(b, s)$, hence $s \in F(p, b) \setminus M(p, b)$. Thus there exists $t \in F(p, b)$ such that $t >_b s$. Consider two cases:

(1) there exists $c \in F(t)$ such that $m(t) = c$. Hence $p(t) = r(c, t)$. We have $t \in F(p, b)$, so $r(b, t) \geq p(t) = r(c, t)$. Hence, by (1.1), $b \geq_t c = m(t)$ and (b, t) is a weakly blocking pair (because $t >_b s = m(b)$), a contradiction.

(2) there is no $c \in F(t)$ such that $m(t) = c$. Hence $p(t) = 0$ and also in this case (b, t) is a weakly blocking pair (we have $b \geq_t \varnothing = m(t)$ and $t >_b s = m(b)$), a contradiction.

Concluding remarks

In our paper we have studied some properties of competitive equilibria in a market model with indivisible goods and unit demand. Our model is based on Gale-Shapley college admissions model, but can be applied to more general situations (for example for studying some auction markets). Studying such models may be very important from the point of view of looking for effective algorithms of computing competitive equilibria, as it was shown in the paper of Chen, Deng and Ghosh [2014]. We have shown that competitive equilibria in the unit demand models can be defined in different ways. The traditional definition, used in literature, needs restrictive assumption of zero pricing of unassigned goods. In our definition of weak equilibrium there is no such assumption, yet, as we have proved, the two important properties of standard definition (maximality and stability with respect to changing the prices of unassigned goods) are still preserved. Thus the weaker notion of equilibrium may be, as we think, better suited for modelling real markets.

In the last section we have related the notion of equilibrium in our model with the classical notion of stable matching of Gale and Shapley. Hence the problem of looking for equilibria for such models can be reduced to the problem of looking for stable matchings, which, in general, is relatively easy task.

It is worth noting that similar results concerning relationships between equilibria and Gale-Shapley stability were proved recently by Hatfield et. al. [2013] and Herings [2015], but our model can not be embedded in these models (in Hatfield et. al. [2013] the utility functions are quasi-linear and in Herings [2015] there are no reservation prices in our sense). Hence it would be interesting to build a general market model which would include as special cases our model and the two above models and for which the relationship between equilibria and Gale-Shapley stability could be proved. But it needs further research.

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PEWNE WŁASNOŚCI RÓWNOWAGI KONKURENCYJNEJ I SKOJARZEŃ STABILNYCH W MODELU RYNKU TYPU GALE'A-SHAPLEYA

Streszczenie: W artykule zbadano pewne własności równowagi konkurencyjnej w modelu rynku typu Gale'a-Shapleya. Podano formalną definicję równowagi, w tym modelu bez założenia o zerowaniu się cen dóbr nieprzydzielonych. Udowodniono też, że alokacje równowagi konkurencyjnej w tym modelu są skojarzeniami silnie stabilnymi w sensie Gale'a-Shapleya (i na odwrót).

Słowa kluczowe: model rynku typu Gale'a-Shapleya, równowaga konkurencyjna, skojarzenie stabilne, dobro niepodzielne, model rynku z jednostkowym popytem.