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# MIDQUOTES OR TRANSACTIONAL PRICES? EVALUATION OF BLACK MODEL ON HIGH-FREQUENCY DATA[1]

**Summary:** The main idea of this research is to check the efficiency of the Black option pricing model on the basis of high frequency emerging market data. However, liquidity constraints – a typical feature of an emerging derivatives market – put severe limits for conducting such a study [Kokoszczyński et al., 2010]. This is the reason we focus on midquotes instead of transactional data being aware that midquotes might not be a proper representation of market prices as probably transactional data are. We compare in this paper our results with the research conducted on high-frequency transactional and midquotes data. This comparison shows that the results do not differ significantly between these two approaches and that Black model with implied volatility (BIV) significantly outperforms other models, especially the Black model with realized volatility (BRV) with the latter producing the worst results.

**Keywords:** option pricing models, midquotes data, realized volatility, implied volatility, microstructure bias.

**JEL Classification:** C61, C22, G14, G15.

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## Introduction

The Black's [1976] futures option pricing model began a new era of futures option valuation theory. The rapid growth of option markets in the 1970s[2] brought soon a lot of data and stimulated an impressive development of research in this area. Soon after this, numerous empirical studies put in doubt basic assumptions of the Black model: they strongly suggest that the geometric Brownian motion is not a realistic assumption for dynamics of underlying prices. Many underlying return series display negative skewness and excess kurtosis [Bates, 2003]. Moreover, implied volatility calculated from the Black-Scholes model often vary with the time to maturity of the options and the strike price [Rubinstein, 1998; Tsiaras, 2009]. These observations drove many researchers to propose new models which relax some of restrictive assumptions of the Black-Scholes model [Broadie, Detemple, 2004; Han, 2008; Mitra, 2009; Garcia, Ghysels, Renault, 2010]. Following Han [2008], we can distinguish several research strands in the literature. The first one engage in extending Black-Scholes-Merton (BSM) framework by incorporating stochastic jumps or stochastic volatility [Hull, White, 1987; Amin, Jarrow, 1992], another concentrates on estimating the stochastic density function of the underlying asset directly from the market option prices [Derman, Kani, 1994; Dupire, 1994] or using other (than normal) distribution of returns of the underlying asset [Corrado, Su, 1996; Rubinstein, 1998]. On the other hand, the Black-Scholes model is still widely used not only as a benchmark in comparative studies testing various option pricing models, but also among market participants. Christoffersen and Jacobs [2004] show that much of its appeal is related to the treatment of volatility – the only parameter of the Black-Scholes model that is not directly observed.

Detailed analysis of literature [Brandt, Wu, 2002; Bates, 2003; Ferreira et al., 2005; Andersen, Frederiksen, Staal, 2007; An, Suo, 2009; Mixon, 2009] seems to suggest that the BSM model with implied volatility calculated on the basis of the last observation performs quite well even when compared with many different pricing models (standard BSM model, BSM with realized volatility, GARCH option pricing models or various stochastic volatility models).

Kokoszczyński et al. [2010] use high-frequency (10-seconds) data for WIG20[3] index options to check whether the same observation applies also to the

[2] The Chicago Board of Options Exchange was founded in 1973 and it adopted the Black-Scholes [Black, Scholes, 1973] model for option pricing in 1975.

[3] The WIG20 is the index of twenty largest companies on the Warsaw Stock Exchange (further detailed information may be found at [www 1]).

Polish market. Their results show that the Black model with implied volatility (BIV) gives the best results, the Black model with historical volatility (BHV) is slightly worse, and the Black models with realized volatility give clearly the worst results. This ranking, based on four different types of error statistics, is rather robust with respect to different times to maturity and moneyness ratios. It is important to notice, that in their research, market prices are represented by midquotes calculated on the basis of the bid and ask quotes. As we know, these quotes do not represent actual prices at which transactions take place.

Nevertheless, most papers we know that test alternative option pricing models and include the Black-Scholes model among models tested therein use bid-ask quotes (midquotes) as they allow to avoid microstructural noise effects [Dennis, Mayhew, 2009]. In addition, Ait-Sahalia and Mykland [2009, p. 592] state explicitly that quotes "contain substantially more information regarding the strategic behaviour of market makers" and they "should be probably used at least for comparison purposes whenever possible". However, Beygelman [2005] and Fung and Mok [2001] argue that a midquote is not always a good proxy for the true value of an option.

Thus, the aim of this article is to investigate whether the conclusions presented in Kokoszczyński et al. [2010] apply to both: transactional data and midquotes.

The structure of this paper has been planned in such a way as to answer the following research questions:

- Can we treat midquotes data as a representation of market prices similar to transactional data in order to reveal specific market features?
- Are there any differences between the results for these two sets of data concerning the efficiency of option pricing models we test?

## 1. Option pricing methodology

The basic pricing model we choose is the Black-Scholes model for futures prices, i.e. the Black model [Black, 1976]. We call it further the BHV model – the Black model with historical volatility. Below are formulas for this model:

$$c = e^{-rT}[FN(d_1) - KN(d_2)], \quad (1)$$

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)], \quad (2)$$

where:

$$d_1 = \frac{\ln(F/K) + \sigma^2 T/2}{\sigma\sqrt{T}}, \tag{3}$$

$$d_2 = \frac{\ln(F/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}, \tag{4}$$

where:
*c* and *p* – respectively valuations of a call and a put option,
*T* – time to maturity,
*r* – the risk-free rate,
*F* – the futures price,
*K* – underlying strike,
σ – volatility of underlying,
*N*(.) – the cumulative standard normal distribution.

One of the most important issues about option pricing is the nature of an assumption concerning the specific type of volatility process. Therefore, we check the properties of the Black model with three different types of volatility estimators: historical volatility, realized volatility, and implied volatility. Detailed formulas for these estimators are presented in Kokoszczyński et al. [2010].

Having these volatility estimators we study several types of option pricing models:

- BHV – the Black model with historical volatility (sigma as standard deviation, $n = 21$ intervals):

$$VAR_{\Delta}^{n} = \frac{1}{(N_{\Delta} * n) - 1} \sum_{t=1}^{n} \sum_{i=1}^{N_{\Delta}} (r_{i,t} - \bar{r})^2, \tag{5}$$

where:
variance of log returns calculated on high frequency data on the basis of last *n* days,
$r_{i,t}$ – log return for *i*-th interval on day *t* with sampling frequency equal to Δ, which is calculated in the following way:

$$r_{i,t} = logC_{i,t} - logC_{i-1,t}\ , \tag{6}$$

$C_{i,t}$ – close price for *i*-th interval on day *t* with sampling frequency equal to Δ,
$N_{\Delta}$ – number of Δ intervals during the stock market session,
*n* – memory of the process measured in days, used in the calculation of respective estimators and average measures,
$\bar{r}$ – average log return calculated for last *n* days with sampling frequency Δ, which is calculated in the following way:

$$\bar{r} = \frac{1}{(N_\Delta * n)} \sum_{t=1}^{n} \sum_{i=1}^{N_\Delta} r_{i,t}, \tag{7}$$

- BRV – the Black model with realized volatility (realized volatility as an estimate of sigma parameter in formulas (3) and (4)); RV (realized volatility) calculated on the basis of observations with several different Δ intervals, where Δ stands for sampling frequency:

$$RV_{\Delta,t} = \sum_{t=1}^{N_\Delta} {r_{i,t}}^2 , \tag{8}$$

- BIV – the Black model with implied volatility (implied volatility as an estimate of sigma in formulas (3) and (4)); IV (implied volatility) calculated for the previous observation, separately for each TTM (time to maturity) and moneyness class, and for both call and put options, hence for 50 different groups).

Initially, we calculate BRV models with four different Δ values: 10s, 1m, 5m, and 15m. Then, we check the properties of averaged RVs with different values of parameter *n* in pricing models. Similarly, like Kokoszczyński et al. [2010], we find no significant differences between different averaged RVs. As a result, we calculate BRV models based only on Δ = 5m interval with different values of averaging parameter (*n* = 5, and 21) and hence, we obtain the following three BRV models: BRV10s (non-averaged one), BRV5m (non-averaged one), BRV5m_5 and BRV5m_21[4].

Finally, in order to verify our research hypothesis, we use root mean squared error (RMSE):

$$RMSE = \sqrt{\frac{1}{N_\Delta n} \sum_{i=1}^{N_\Delta n} (Black_i - close_i)^2}, \tag{9}$$

where:
$close_i$ – the option price (midquote or last observed transaction price) for the *i*-th interval,
$Black_i$ – the Black model price (BHV, BRV or BIV) for the *i*-th interval,
$N_\Delta$ – number of Δ intervals during the stock market session,
RMSE – calculated for all models, for different TTM and MR classes, and for both call and put options.

[4] It is common approach in financial research to set the interval between 5 minutes and 15 minutes because they constitute the good trade-off between the non-synchronous bias and other microstructure biases.

## 2. Data and description of volatility processes

### 2.1. Data description

In empirical analysis we apply transaction data for the WIG20 index options and WIG20 futures contracts, obtained from the DM BOŚ provider. The sample covers the period from January 2nd, 2008 to June 20th, 2008. We have aggregated the options transactional prices data from the original frequency of 1 second into 1 minute interval, and use the WIG20 futures prices data that have 10 second interval[5]. In each trading day, sessions begin at 9 am and finish at 4:30 pm[6]. Hence, we have 53 218 observations (118 trading days, 451 observations for each trading session). The risk free interest rate is approximated by the WIBOR interest rate, also converted into 1-minute intervals.

The whole data set comprises transaction prices for 65 call index options and 63 put index options expiring in March, June, and September 2008 (C, F and I series for call options, and O, R and U series for put options). In order to present the results of analysis, we order them according to:

- 2 types of options (call and put),
- 5 classes of moneyness ratio (MR), for call options: deep OTM (0-0.85), OTM (0.85-0.95), ATM (0.95-1.05), ITM (1.05-1.15) and deep ITM (1.15+), and for put options in the opposite order[7],
- 5 classes for time to maturity (TTM): (0-15 days], [16-30 days], [31-60 days], [61-90 days], [91+ days).

This categorization allows us to compare different pricing models along numerous dimensions.

### 2.2. Description of volatility processes

We consider three different volatility measures: historical, realized and implied volatility. Obviously, this is the reason for differences between theoretical option prices we compare.

5 We do not aggregate WIG20 futures data into 1-minute intervals, because we also want to include RV estimators with Δ parameter of frequency higher than 1 minute.

6 Actually, the continuous trading stops at 4:10 p.m. Between 4:10 pm and 4:20 pm close price is settled and then, till 4:30 pm investors can trade only on the basis of close price.

7 Moneyness ratio is usually calculated, according to the following formula:

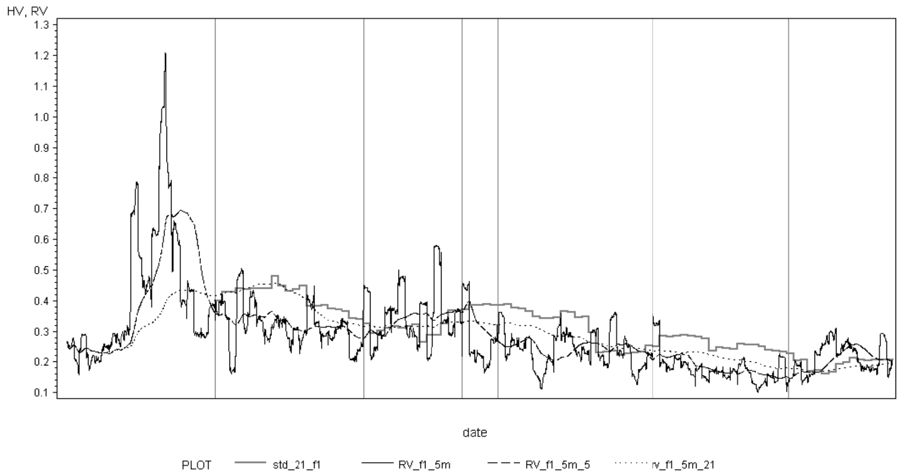
$$\text{moneyness ratio} = \frac{S}{K / e^{rT}} = \frac{F}{K}$$

where: $K$ is the option strike price, $S$ – the price of underying, $F$ – the futures price of underlying, $r$ – the risk free rate, and $T$ – time to maturity.

In the case of the historical volatility estimator $N_{\Delta} = 1$ for every $r_{i,t}$ (daily log returns) and $C_{i,t}$ in formulas (5) to (8). Moreover, we use the constant value of parameter $n = 21$, as we want to reflect the historical volatility from the last trading month.

Realized volatility was initially calculated for Δ = 10s, 1m, 5m and 15m. However, Kokoszczyński et al. [2010] show that differences between theoretical options prices from the Black model with RV calculated with these four Δ parameters are negligible. Therefore, we present our results for RV calculated only with Δ = 10s and Δ = 5m[8]. However, the procedure of averaging has been applied only for 5-minute interval and *n* days, where *n* = 5 and 21.

Figure 1 presents realized volatility compared to historical volatility time series. The distinguishing fact is that the not-averaged RV time series is much more volatile than the averaged RV or HV time series. Obviously, such high volatility of volatility can strongly influence theoretical prices of the BRV model and its stability over time. One can thus expect that in periods of high returns volatility the BRV model with non-averaged RV estimator may produce high pricing errors.




Note:
The volatility time series cover the data period between January 2nd, 2008 and June 19th, 2008. Vertical lines represent end of month and additionally the day of March 20th, when option series C (call) and O (put) matured. Std_21_n – stands for HV calculated based on 5m intervals with memory equaled 21 days, RV_n_5m, RV_n_5m_r, RV_n_5m_21 –stands for RV calculated based on 5m intervals with memory equaled 1, 5, and 21 days.

**Figure 1.** Historical and realized volatility (5m, 5m_5, 5m_21)

Source: Own calculations.

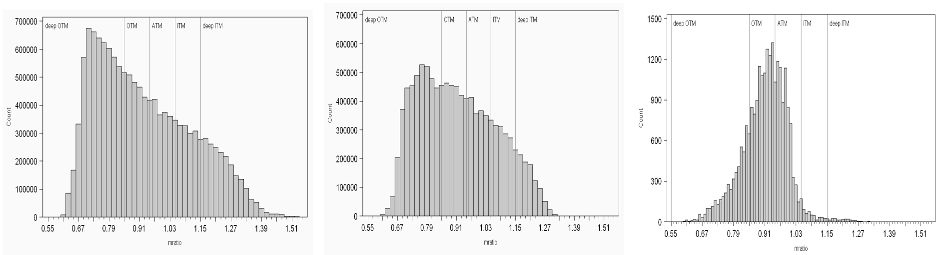
[8] As mentioned before, although transactional data is of 1-minute frequency, we have decided to include RV estimator also with Δ higher than 1 minute (seconds or tick).

## 3. The liquidity issue

As we mentioned earlier, liquidity constraints – a typical feature of an emerging derivatives market – put severe limits for conducting such a study as we present here. It was the reason why Kokoszczyński et al. [2010] conducted their research using midquotes data. Therefore, currently we verify their previous results using transactional data for the same time period.

Figure 2 and Figure 3 present the comparison of midquotes and transactional data sets with respect to the possibility of trades. When we look at Figure 2 and Figure 3, we observe considerable difference between the opportunity to trade indicated in available strike prices or active midquotes and the actual trades revealed by transactional data. The similar patterns are observed in the case of call and put options and that is the additional confirmation for the results presented in the earlier figures.

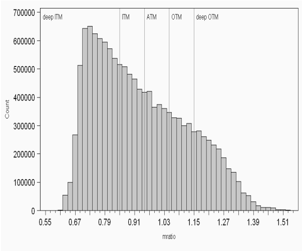
The most important outcome from the liquidity analysis is the major difference in the trade volume between midquotes and transactional data. We can see that the number of actual trades is on average less than 0.2% of potential trades indicated by active midquotes. It obviously confirms the low liquidity phenomenon of emerging markets and it is the reason why we have decided to conduct addititonal study applying transactional data in order to verify results obtained for midquotes data.



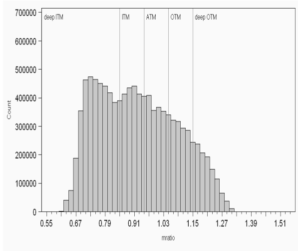

| Available strike prices indicate options which were introduced by the WSE on the market. | Active midquotes indicate options which were quoted in the sample period. | Transactional data; the nominal scale on vertical axis was tranformed. |
|---|---|---|

**Figure 2.** Moneyness ratio histogram for call options with respect to available strike prices, active midquotes and transactional data

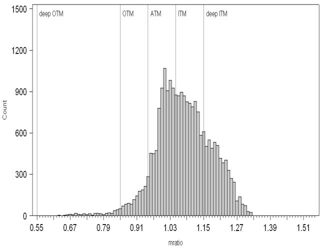
Source: Own calculations.




Available strike prices indicate options which were introduced by the WSE on the market.




Active midquotes indicate options which were quoted in the sample period.




Transactional data; the nominal scale on vertical axis was tranformed.

**Figure 3.** Moneyness ratio histogram for put options with respect to available strike prices, active midquotes and transactional data

Source: Own calculations.

## 4. Results

RMSE is calculated for all pricing models (BRV10s, BRV5m, BRV5m_5, BRV5m_21, BHV, BIV, and additionally for the BRV model with different values of parameter *n*) which are divided into 5 TTM classes and 5 MR classes. Frequencies of predicted premiums for each model is presented in Table 1.

**Table 1.** Number of predicted premiums for different classes of MR and TTM for BRV model*

| Option | Moneyness | 0-15 days | 16-30 days | 31-60 days | 61-90 days | 91+ days | *Total* |
|---|---|---|---|---|---|---|---|
| CALL | deep OTM | 205 | 304 | 1 726 | 1 670 | 985 | ***4 890*** |
| CALL | OTM | 1 586 | 2 037 | 3 280 | 2 285 | 1 161 | ***10 349*** |
| CALL | ATM | 3 403 | 1 235 | 1 437 | 771 | 409 | ***7 255*** |
| CALL | ITM | 85 | 35 | 165 | 157 | 59 | ***501*** |
| CALL | deep ITM | 15 | 26 | 81 | 31 | 36 | ***189*** |
| ***Total Call*** | | **5 294** | **3 637** | **6 689** | **4 914** | **2 650** | ***23 184*** |
| PUT | deep OTM | 368 | 857 | 2 134 | 1 014 | 1 011 | ***5 384*** |
| PUT | OTM | 1 615 | 1 170 | 2 345 | 1 694 | 917 | ***7 741*** |
| PUT | ATM | 3 416 | 1 215 | 1 559 | 1 005 | 423 | ***7 618*** |
| PUT | ITM | 450 | 144 | 283 | 215 | 107 | ***1 199*** |
| PUT | deep ITM | 19 | 8 | 48 | 61 | 102 | ***238*** |
| ***Total Put*** | | **5 868** | **3 394** | **6 369** | **3 989** | **2 560** | ***22 180*** |
| ***Total Call and Put*** | | ***11 162*** | ***7 031*** | ***13 058*** | ***8 903*** | ***5 210*** | ***45 364*** |

* 45.9 thou. for BIV, and 37 thou. for BHV

Source: Own calculations.

Further results are divided among two subsections containing the description for HF transactional data results, the comparison between HF transactional data and midquotes results.

## 4.1. The description of results for HF transactional data

The discussion of results for HF transactional data is based on two-dimensional charts presented as panels containing five or six boxes where we show RMSE for all models, all MR and TTM classes. Each chart is scaled with global minima and maxima and that enables simple and reliable comparison of presented results. Figures 4-6 present error statistics for call and put options separately, with individual boxes for different MR, albeit for all TTM and all models in one box.

Figure 4 shows RMSE statistics for call options. The best results are observed for the BIV model, but we observe results as good as those for each model with TTM equal to 0-30 days. Analysing the results for the remaining values of TTM we see gradual decrease of RMSE statistics while moving from the left hand side of each chart (model 1) to its right hand side (model 6). These observations confirm once again the ranking of our models: from the BIV model through the BHV and the BRV5m_21 ones to the non-averaged BRV model.

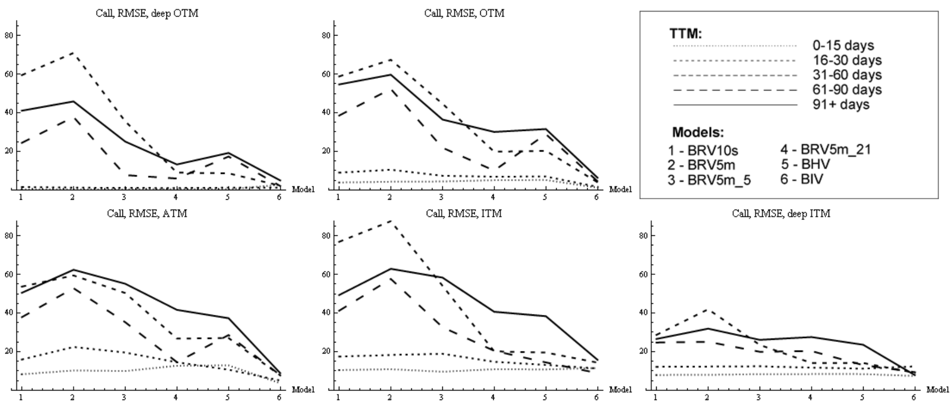



**Figure 4.** RMSE statistics for call options for all MR with respect to different pricing models and TTM

Source: Own calculations.

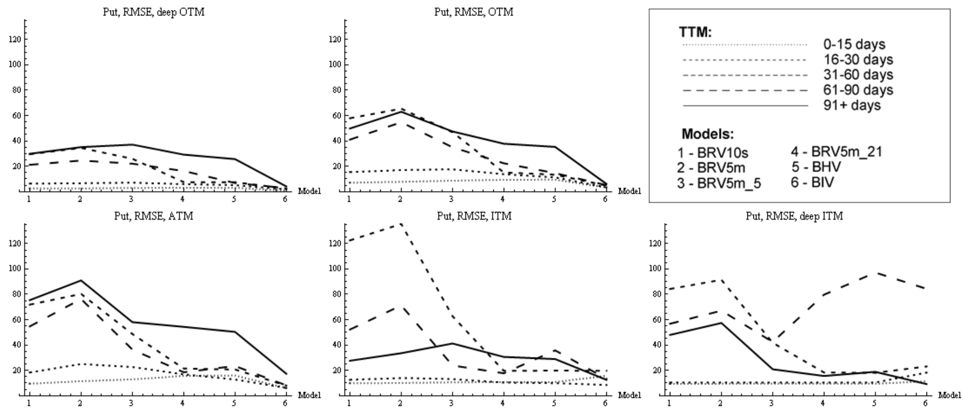



**Figure 5.** RMSE statistics for put options for all MR with respect to different pricing models and TTM

Source: Own calculations.

Figure 5 presents RMSE statistics for put options and shows almost the same results as those for call options. The errors gradually decrease from the left-hand side to the right-hand side with practically identical error values for all models with TTM equal to 0-30 days. The only exception we observe here are high values of error statistics when MR is deep ITM and TTM equals 61-90 days for the BRV5_21, the BHV and the BIV models. The reason for so untypical observation could be a very low number of transaction for deep ITM put options with high TTM values. Nevertheless, these results confirm the ranking of models (model 6 dominates model 1).

## 4.2. Comparison of results for midquotes and transactional data

One of the goals of this paper is to answer the question, how firm are our conclusions concerning the option market that we have got using midquotes data. To check this we repeat the previous study of Kokoszczyński et al. [2010] using now transactional data. After discussion of results for the latter in section 4.1 the comparison of results for two different data sets will be presented in this section.

Figure 6 with RMSE statistics for call options does not reveal any significant differences when compared with Figure 4, both with respect to the ranking of models and to errors dependence on TTM or MR values.

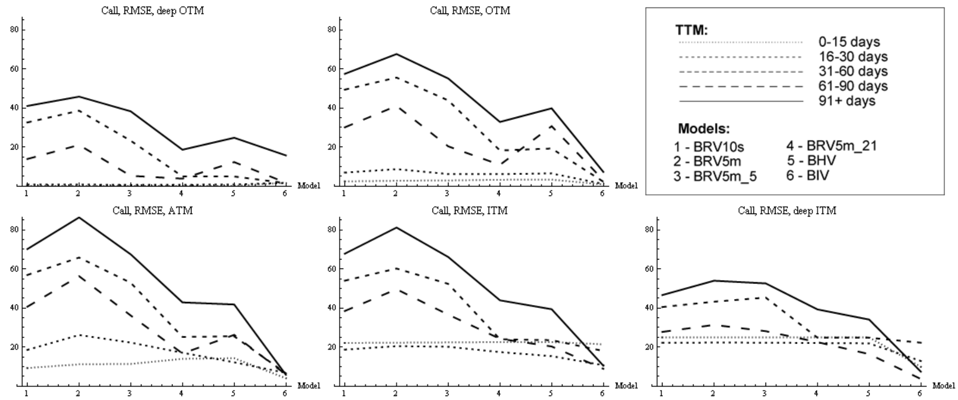



**Figure 6.** RMSE statistics for call options for all MR with respect to different pricing models and TTM. Midquotes data

Source: Own calculations.

The comparison of the results for put options (Figure 7 for midquotes data vs. Figures 5 for transactional data) does not add any new insights to the conclusions based on the results for call options. RMSE statistics for transactional data present the same picture as for midquotes data.

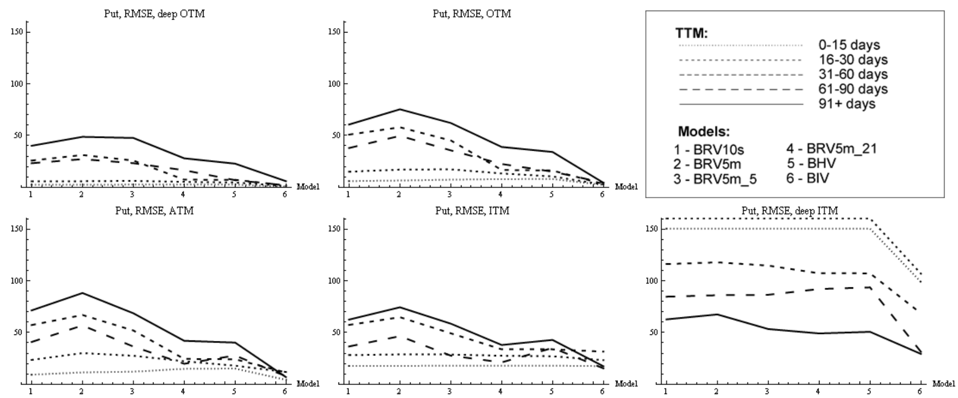



**Figure 7.** RMSE statistics for put options for all MR with respect to different pricing models and TTM. Midquotes data

Source: Own calculations.

This brief comparison informs us that we do not observe any important differences between the results for midquotes and transactional data. Therefore, we can use the former in our research for countries where liquidity issue (which is usually the characteristic of emerging country) plays an important role. Two sets of data may give different outcomes with respect to outliers which can distort data in a different manner because the number of observations for midquotes and transactional data is usually not the same.

## Conclusions and further research

We decided to evaluate and develop further the study by Kokoszczyński et al. [2010] based on WIG20 option index data for the first half of 2008 year and to check if those results were still valid not only for midquotes data but also for transactional data. Furthermore, we presented the analysis of liquidity for option market in order to better understand different behaviour of option market within various classes of TTM and MR.

First of all, the results for transactional data do not differ significantly from the results based on midquotes data. The sequence of models, from the most efficient to the least one, is as follows: BIV, BHV, BRV5m_21, BRV5m_5, BRV5m, BRV10s. Moreover, the variability of observed value of analysed error statistics when we move from model 1 (BRV10s) to model 6 (BIV) become much lower what additionally confirms our previous results concerning the efficiency of these models. Focusing on parameter $n$ and only on BRV models we observe that the lowest value of error is obtained for the highest tested $n = 21$, what confirms our initial hypothesis that non-averaged value of RV estimator is not the best choice when we consider the efficiency of option pricing model. On the other hand these results do not give us the definite answer to the question, what is the optimal value of parameter $n$. Further research should address this issue. Next, we observe the clear relation between model error and TTM, and model error and moneyness ratio (for call and put options): high error values for low TTM and moneyness ratios, and best fit for high TTM and moneyness ratios. All these outcomes confirm our initial hypothesis that midquotes are a proper representation of market prices and can be used in similar studies, especially in case of low liquidity markets.

Analysing liquidity issues we can see several interesting features of midquotes and especially of transactional data. First of all, the volume of call and puts is focused on ATM, OTM and deep OTM options with hardly any volume for deep ITM and ITM options. What is more important, the behaviour of this characteristic is robust for transactional data and depends on the actual market fluctuations for midquotes data. Secondly, the volume of turnover focuses around ATM options, indicating that when we consider the value of transactions the highest liquidity is observed for ATM options. Thirdly, we observe similar pattern for number of open positions as described for their volume. Fourthly, no matter which characteristic do we choose, the liquidity is significantly higher for put options. However, we are aware of the fact that the last observation could result from the sharp downward movement of the market in the time of research and high demand for put options for hedging purposes.

More generally, our results seem to confirm that the nature of data used for studies of option models – midquotes or transactional ones – does not play the important role in determining results one gets. Another observation, i.e. how important are liquidity issues for patterns, we get comparing performance of various option pricing models, should be studied further.

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## KWOTOWANIA MID OPCJI CZY ICH CENY TRANSAKCYJNE? EWALUACJA MODELU BLACKA NA DANYCH WYSOKIEJ CZĘSTOTLIWOŚCI

**Streszczenie:** Głównym celem artykułu jest weryfikacja efektywności modelu Blacka wyceny opcji na podstawie danych wysokiej częstotliwości dla rynku rozwijającego się. Ograniczenia dotyczące płynności opcji – typowa charakterystyka instrumentów pochodnych na rynkach rozwijających się – stanowią jednak istotne ograniczenie dla takiego badania [Kokoszczyński et al., 2010]. Niska płynność jest jedną z przyczyn, dla których wykorzystuje się kwotowania mid zamiast danych transakcyjnych ze świadomością, że dane transakcyjne mogą być lepszą reprezentacją aktualnego stanu rynku na danym instrumencie finansowym. W badaniu porównano obliczenia przeprowadzone na danych wysokiej częstotliwości dla cen transakcyjnych i kwotowań mid. Porównanie to pokazuje, że rezultaty praktycznie nie różnią się dla tych dwóch różnych danych wejściowych i model Blacka ze zmiennością implikowaną (BIV) osiąga znacznie lepsze wyniki od pozostałych modeli, szczególnie w porównaniu z modelem Blacka ze zmiennością zrealizowaną (BRV).

**Słowa kluczowe:** modele wyceny opcji, średnie kwotowania opcji, zmienność zrealizowana, zmienność implikowana, mikrostruktura rynku.